Online Appendix to "Specification and Negotiation in Incomplete Contracts: An Econometric Analysis"

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1 Non-identification without Normalization

For a single-agent model with a non-additive monotone outcome function, Matzkin (2003) showed some scale normalization of unobserved shocks is necessary for nonparametric identification.[1] In comparison, we show that normalizing the distribution of the buyer's private signals is necessary for identifying our game-theoretic model, where the vector of outcomes (V, D, Y, X, X^*) reported are rationalized by strategic interaction between contractors and the buyer in monotone psPBE. To do so, we need to use a two-step approach that is qualitatively different from Matzkin (2003). We first characterize how players' equilibrium strategies vary with model parameters. Then we show how distinct sets of model parameters could generate the same distribution of outcome (V, Y, D, X, X^*) in equilibrium.

We say two models are *observationally equivalent* if they imply the same distribution of V, Y, D, X and X^* in symmetric psPBE. Suppose the actual data-generating process is $\theta \equiv \{\gamma, \pi, a, F_{C|X}, F_{X^*,\tilde{X}}\}$. Consider an alternative model $\theta_0 \equiv \{\gamma_0, \pi_0, a_0, H_{C|X}, H_{X^*,\tilde{X}}\}$ such that (i) $a = a_0$, $\pi = \pi_0$ and $\gamma = \gamma_0$; (ii) $F_{C|X=x} = H_{C|X=x}$ for all $x \in \mathcal{X}$; and (iii) $F_{X^*,\tilde{X}}(x, \tilde{x}) = H_{X^*,\tilde{X}}(x, h(\tilde{x}))$ for all $x, \tilde{x}$ and some increasing and differentiable function $h : \widetilde{\mathcal{X}} \to \mathcal{T} \subset \mathbb{R}$. In other words, all structural elements in θ and θ_0 are identical, except that the procurer's private signal in θ_0 is an increasing and differentiable transformation of that in θ. Note the support of the private signal in θ_0 (denoted $\mathcal{T}$) is allowed to differ from that in θ.

Lemma 1 (Observable Equivalence) *Suppose θ and θ_0 are two data-generating processes satisfying (i)-(iii) in the preceding paragraph, then θ and θ_0 are observationally equivalent.*

[1] Matzkin (2003) considered a nonparametric model $Y = m(X, \epsilon)$ where m is monotone in ϵ, and ϵ is independent from X. Lemma 1 in Matzkin (2003) established that, without further restrictions on m, the model is observationally equivalent to another model $Y = \widetilde{m}(X, \tilde{\epsilon})$ where $\tilde{\epsilon}$ is any monotone transformation of ϵ.

Proof of Lemma 1. The first step is to relate the equilibrium strategy in θ to that in θ_0. Let $\alpha(\cdot;\theta) : \widetilde{\mathcal{X}} \to \mathcal{X}$ denote the procurer strategy under θ in a symmetric monotone psPBE; and likewise define $\alpha(\cdot;\theta_0)$. Let $\alpha, \dot{\alpha}$ be shorthand for $\alpha(\tilde{x})$ and $\alpha'(\tilde{x})$ respectively. Recall that in equilibrium the procurer's strategy $\alpha(\cdot;\theta)$ solves the ordinary differential equation (ODE):

$$\dot{\alpha} = \frac{\gamma\Lambda_2(\alpha,\tilde{x};\theta)}{\sigma'(\alpha;\theta) - \pi'(\alpha) - \Lambda_1(\alpha,\tilde{x};\theta)} \equiv \varpi(\alpha,\tilde{x}) \tag{1}$$

for an initial condition $\alpha(\tilde{x}_l;\theta) = x_l^o$, where μ, δ, σ are defined above. We highlight the dependence of $\Lambda_1, \Lambda_2, \sigma$ on θ throughout this subsection. While Λ_1 and Λ_2 depend on $\{\gamma, \pi, a, F_{X^*,\tilde{X}}\}$, $\sigma : \mathcal{X} \to \mathbb{R}$ only depends on $F_{C|X}$ in θ. Likewise, $\alpha(\cdot;\theta_0)$ solves an ODE similar to (1), only with θ therein replaced by θ_0:

$$\dot{\alpha}_0 = \frac{\gamma\Lambda_2(\alpha_0,\tilde{x};\theta_0)}{\sigma'(\alpha_0;\theta) - \pi'(\alpha_0) - \Lambda_1(\alpha_0,\tilde{x};\theta_0)} \tag{2}$$

and a corresponding initial condition. By construction, $F_{C|X} = H_{C|X}$ and $F_{X^*|\tilde{X}=\tilde{x}} = H_{X^*|\tilde{X}=h(\tilde{x})}$ for all $\tilde{x} \in \mathcal{X}$. Hence $\sigma'(\cdot;\theta) = \sigma'(\cdot;\theta_0)$ and

$$\Lambda_k(\cdot, h^{-1}(t);\theta) = \Lambda_k(\cdot, t;\theta_0), k = 1,2 \tag{3}$$

for any $t \in \mathcal{T}$. Substitute (3) into the ODE for θ_0 in (2) gives:

$$\dot{\alpha}_0 = \frac{\gamma\Lambda_2(\alpha_0, h^{-1}(t);\theta)}{\sigma'(\alpha_0;\theta) - \pi'(\alpha_0) - \Lambda_1(\alpha_0, h^{-1}(t);\theta)} \left(\frac{dh^{-1}(t)}{dt}\right) \tag{4}$$

for any $t \in \mathcal{T}$. To establish the relation between $\alpha(\cdot;\theta)$ and $\alpha(\cdot;\theta_0)$, we need to show the following auxiliary lemma.

Lemma 2 *Let $\zeta : \mathcal{T} \to \mathcal{X}$ be an increasing differentiable function, where $\mathcal{T}$, $\mathcal{X}$ are open sets on $\mathbb{R}$. If $\alpha(\tilde{x})$ solves the ODE "$\dot{\alpha} = \varpi(\alpha,\tilde{x})$, $\tilde{x} \in \mathcal{X}$" with an initial condition $\alpha(\tilde{x}_l) = x_l^o$, then $\alpha_0(t) \equiv \alpha(\zeta(t))$ solves "$\dot{\alpha}_0 = \varpi(\alpha_0,\zeta(t))\zeta'(t)$, $t \in \mathcal{T}$" with an initial condition $\alpha_0(\zeta^{-1}(\tilde{x}_l)) = x_l^o$.*

Proof of Lemma 2. By the definition of $\alpha(\tilde{x})$ as a solution to the first ODE, we have $\alpha'(\tilde{x}) = \varpi(\alpha(\tilde{x}),\tilde{x})$ for all $\tilde{x} \in \mathcal{X}$. If $\alpha_0(t) \equiv \alpha(\zeta(t))$ for all $t \in \mathcal{T}$, then $\alpha_0'(t) = \alpha'(\zeta(t))\zeta'(t)$ for all $t \in \mathcal{T}$. Combining the two equalities, we have $\alpha_0'(t) = \varpi(\alpha(\zeta(t)),\zeta(t))\zeta'(t)$ for all $t \in \mathcal{T}$. With $\alpha_0(t) \equiv \alpha(\zeta(t))$, this equality can be written as $\alpha_0'(t) = \varpi(\alpha_0(t),\zeta(t))\zeta'(t)$ for all $t \in \mathcal{T}$. Finally, note $\alpha(\tilde{x}_l) = x_l^o$ and $\alpha_0(t) = \alpha(\zeta(t))$ implies $\alpha_0(\zeta^{-1}(\tilde{x}_l)) = \alpha(\tilde{x}_l) = x_l^o$. This proves the lemma. $\square$

An application of Lemma 2 to (4) with $\zeta \equiv h^{-1}$ and $\varpi(\alpha,\tilde{x})$ defined on the r.h.s. of (1) implies the procurer equilibrium strategies under θ_0 and θ are related as $\alpha(t;\theta_0) = \alpha(h^{-1}(t);\theta)$ for all $t \in \mathcal{T}$.

Next, note that the joint distribution of (X^*, X) evaluated at (x^*, x) according to θ is $F_{X^*,\tilde{X}}(x^*, \alpha^{-1}(x;\theta))$ while that under θ_0 is $H_{X^*,\tilde{X}}(x^*, \alpha^{-1}(x;\theta_0))$. The relation between $\alpha(\cdot;\theta)$ and $\alpha(\cdot;\theta_0)$ implies that $\alpha^{-1}(x;\theta_0) = h(\alpha^{-1}(x;\theta))$ for all $x \in \mathcal{X}$. Hence

$$H_{X^*,\tilde{X}}(x^*, \alpha^{-1}(x;\theta_0)) = H_{X^*,\tilde{X}}(x^*, h(\alpha^{-1}(x;\theta))) = F_{X^*,\tilde{X}}(x^*, \alpha^{-1}(x;\theta)) \tag{5}$$

where the second equality is due to the specified relation between $F_{X^*,\tilde{X}}$ and $H_{X^*,\tilde{X}}$. Therefore the two joint distributions of (X^*, X) in equilibrium under θ and θ_0 are identical. Let $\phi(x, x^*) \equiv \pi(x^*) - \pi(x)$ and $\phi_0(x, x^*) = \pi_0(x^*) - \pi_0(x)$. Then that $a = a_0$, $\pi = \pi_0$ and $\gamma = \gamma_0$ in θ and θ_0 imply $\gamma\phi + (1-\gamma)a = \gamma_0\phi_0 + (1-\gamma)a_0$ for all x, x^*. Furthermore, a pair (X, X^*) leads to positive net surplus $\phi - a$ under θ if and only if it does so under θ_0. Hence the two models θ and θ_0 imply the same probability of $D = 1$ given (X, X^*), and the same distribution of Y given $D = 1$ and (X, X^*) in equilibrium.

Finally, the distribution of prices quoted by contractors at equilibrium under θ is determined by the distribution of costs $F_{C|X}$ as well as the hold-up $\delta(X; \alpha(\cdot;\theta)) \equiv \mathbb{E}[\gamma s_+(X, X^*) \mid X; \alpha(\cdot;\theta)]$. By construction, $F_{C|X} = H_{C|X}$ and $\gamma(\phi - a) = \gamma_0(\phi_0 - a_0)$. Furthermore, the argument above already shows that the joint distribution of (X, X^*) in psPBE is identical under θ and θ_0. It then follows that θ and θ_0 can generate the same joint distribution of V, Y, D, X and X^* in psPBE. $\square$

2 Incomplete Contracts in Bilateral Negotiation

In practice, contractual incompleteness also exist in bilateral contracts between a procurer and a single contractor. In this case, the sole contractor does not face any peer competition in the pre-contract stage, and the initial payment from the procurer is also determined via a direct negotiation between both parties. Such a situation could arise from some matching process whereby the contract is highly customized to match the procurer's special need, or simply as a result of the contractor's monopoly power on the market. We show that the identification results from Section 3 can be extended to such a bilateral-negotiation case.

In the first stage, the procurer observes a private signal $\widetilde{X} \in \widetilde{\mathcal{X}}$, and announces a specification $X \in \mathcal{X}$ strategically to maximize his total ex ante surplus from the contract. The contractor is notified of X and then negotiates with the procurer to set a first-stage payment $\psi(X)$ through a Nash bargaining process, where ψ is differentiable over $\mathcal{X}$. The cost for delivering the contract at the initial specification X is $c(X)$. In the second stage, a feasible new design X^* is realized, leading to an incremental surplus $\phi(X, X^*) \equiv \pi(X^*) - \pi(X)$ with an incremental cost $a(X, X^*)$, where $\pi(X^*)$ and $\pi(X)$ are respectively the social surplus that accrue to the buyer under the design X^* and X. Assume both π and a are bounded and continuously differentiable over their respective domains. The new design will supplant the initial specification if it yields a positive net incremental surplus $s \equiv \phi - a > 0$, where we suppress the arguments X and X^* for ease of notation. In this case, the contractor then re-negotiates with the procurer via Nash Bargaining to set an additional transfer Y from the procurer. The contractor pays incremental costs up front and the surplus is accrued to the procurer. The bargaining power is fixed and is common knowledge throughout the two stages: $\gamma \in (0,1)$ for the contractor and $1-\gamma$ for the procurer. The other elements π, a, c and $F_{X^*|\tilde{X}}$ are also common knowledge for both parties.

With $\psi(x)$ already determined in the first stage, the additional transfer from the procurer to the contractor in the second stage is characterized by the solution to a Nash Bargaining process

$$y(x,x^*) = \arg\max_t [\psi(x) + t - c - a(x,x^*) - d_c]^\gamma \left[\pi(x^*) - \psi(x) - t - d_p\right]^{1-\gamma}$$

where $d_c \equiv \psi(x) - c$ and $d_p \equiv \pi(x) - \psi(x)$ are the disagreement values for the contractor and the procurer, respectively, where c is the ex post cost for delivering the contract under the initial specification x. The feasible new specification x^* is realized in the second stage and therefore is taken as an ingredient for the Nash Bargaining problem. It then follows that the transfer is given by

$$y(x,x^*) = \gamma\phi(x,x^*) + (1-\gamma)a(x,x^*) \tag{6}$$

for all x, x^* when the new design is adopted (i.e. $s(x,x^*) > 0$); and the transfer is zero otherwise. This takes the same form as in the model with pre-contractual competition in auctions. We maintain that the negotiated price under an initial specification x in the first stage is the solution to a Nash Bargaining problem with the feasible set given by the ex ante surplus from the contract, and thus satisfies:

$$\gamma \left\{ \pi(x) - c(x) + E[s_+(x,x^*)|\widetilde{X} = \tilde{x}] \right\} = \psi(x) - c(x) + \gamma E[s_+(x,x^*)|\widetilde{X} = \tilde{x}].$$

The expression between the braces on the left-hand side is the ex ante net surplus of the contract; the right-hand side is the ex ante payoff for the contractor. This implies:

$$\psi(x) = \gamma\pi(x) + (1-\gamma)c(x). \tag{7}$$

A procurer chooses an optimal initial specification to maximize its ex ante payoff:

$$\alpha^*(\tilde{x}) = \arg\max_{x\in\mathcal{X}} \left\{ \pi(x) + \mu(x,\tilde{x}) - \psi(x) \right\}, \tag{8}$$

where $\mu(x,\tilde{x}) \equiv (1-\gamma)E[s_+(x,x^*)|\widetilde{X} = \tilde{x}]$ as in the case with pre-contractual competition, and we assume $\mu(x,\tilde{x})$ is continuous in $\tilde{x}$. It is straightforward to to show the existence of a strictly increasing strategy of the procurer $\alpha^*(\cdot)$ by tailoring the proof in Appendix B, thus we omit the details. The hold-up on the procurer is similarly defined as $\delta(x) = \gamma E[s_+(x,x^*)|X = x]$, where the expectation is taken with respect to the conditional distribution $F_{X^*|X=x} = F_{X^*|X=\alpha^*(\tilde{x})}$.

In a typical bilateral contract with incompleteness, data report the same set of observables as the procurement auctions with incomplete contracts: the contract price in the first-stage (denoted V), initial contract designs X, as well as new designs X^* and negotiated transfers Y (if the new design is adopted). The model primitives include $\{\pi, a, \gamma, c, F_{X^*,\tilde{X}}\}$.

We follow the identification argument in the main text to identify the model of bilateral contracts. First of all, we impose the same normalization restriction to the distribution of $\tilde{x}$ as in Section 3.1 to identify $F_{X^*,\tilde{X}}$. Following the argument in the main text, we can identify the surplus function π under Assumption 1 as well as a location normalization. It is worth noting that to identify π, the first-order-condition for an interior solution $\alpha^*(\tilde{x})$, i.e., (7) in the main text needs be replaced by

$$\psi'(\alpha^*(\tilde{x})) = \pi'(\alpha^*(\tilde{x})) + \frac{\partial}{\partial x}\left[\int_{\omega(x)} (1-\gamma)s(x,t)dF_{X^*|\tilde{X}=\tilde{x}}(t)\right]_{x=\alpha^*(\tilde{x})}, \tag{9}$$

where $\omega(x)$ is analogously defined as in the main text, i.e., for each $x \in \mathcal{X}$, $\omega(x) \equiv \{x^* \in \mathcal{X} : s(x, x^*) > 0\}$.

We further impose Assumptions 2-3 to identify a and γ as in Corollary 1. Then the hold-up $\gamma E[s_+(x, x^*)|X = x]$ is identified. Finally, since $\psi(x)$ is directly recoverable from the data, it then follows from (7) that the contractor's cost function $c(X)$ is also identified as $c(X) = [\psi(X) - \gamma\pi(X)]/(1-\gamma)$. We summarize the identification results in the following corollary while omitting the proof.

Corollary 2 *(Theorem 1) Suppose that Assumptions 1-3 hold. Then (a) γ is identified; (b) $a(x, x^*)$ is identified for all $x \in \mathcal{X}_e$ and $x^* \in \omega(x)$, and $\pi(x)$ is identified for all $x \in \mathcal{X}_s$; and (c) the hold-up $\delta(x)$ and the cost function c(x) are identified for all $x \in \mathcal{X}_e$.*

3 Identification of the Parametric Model

This model in Section 5 accommodates both contract and contractor heterogeneity and includes additive structural errors ε, η. The following proposition establishes the identification of this parametric model under mild support conditions of the explanatory variables. We assume structural parameters are non-zero (that is, $\theta_r \neq 0$ for $r = 1, 2, ..., 6$, $\sigma \neq 0$ and $\lambda \neq 0$.)

Proposition D.1. *In the model (9)-(10) with the specification (11)-(12), the structural parameters $\{\theta_j\}_{j=1,...,6}, \varrho, \rho$ and σ^2 are identified if the support of $[(x^* - x), (x^* - x) \times job, x^{*2} - x^2, (x^* - x)^2, w]$ has full rank.*

We now sketch a proof of this identification result. To simplify presentation, we first condition on the other explanatory variables in w (other than job) and suppress them in the notation below. Let γ_k be shorthand for the bargaining power when $job = k \in \{0, 1\}$. To begin with, expand the latent variable $\gamma_k\phi + (1-\gamma_k)a + \varepsilon$ in the outcome equation (9) into:

$$\begin{aligned} & \gamma_k[\theta_1(x^* - x) + \theta_2(x^* - x)k + \theta_3(x^{*2} - x^2)] \\ & +(1-\gamma_k)[\theta_0 + \theta_4(x^* - x) + \theta_5(x^* - x)k + \theta_6(x^* - x)^2] + \varepsilon \\ = \; & \beta_{k,0} + \beta_{k,1}(x^* - x) + \beta_{k,2}x^2 + \beta_{k,3}x^{*2} + \beta_{k,4}(x^* - x)k + \beta_{k,5}x^*x + \varepsilon \end{aligned}$$

where

$$\begin{aligned}
\beta_{k,0} &\equiv (1-\gamma_k)\theta_0;\ \beta_{k,1} \equiv \gamma_k\theta_1 + (1-\gamma_k)\theta_4;\ \beta_{k,2} \equiv [(1-\gamma_k)\theta_6 - \gamma_k\theta_3]; \\
\beta_{k,3} &\equiv \gamma_k\theta_3 + (1-\gamma_k)\theta_6;\ \beta_{k,4} \equiv [\gamma_k\theta_2 + (1-\gamma_k)\theta_5];\ \beta_{k,5} \equiv -2(1-\gamma_k)\theta_6
\end{aligned}$$

for $k \in \{0, 1\}$.

Also we can write the latent index in the selection equation (10) as

$$\begin{aligned}
& \theta_1(x^* - x) + \theta_2(x^* - x)k + \theta_3(x^{*2} - x^2) - [\theta_0 + \theta_4(x^* - x) + \theta_5(x^* - x)k + \theta_6(x^* - x)^2] \\
= \ & \tilde{\beta}_0 + \tilde{\beta}_1(x^* - x) + \tilde{\beta}_2(x^* - x)job + \tilde{\beta}_3(x^{*2} - x^2) + \tilde{\beta}_4(x^* - x)^2 + \eta
\end{aligned}$$

where

$$\tilde{\beta}_0 \equiv -\theta_0;\ \tilde{\beta}_1 \equiv \theta_1 - \theta_4;\ \tilde{\beta}_2 \equiv \theta_2 - \theta_5;\ \tilde{\beta}_3 \equiv \theta_3;\ \tilde{\beta}_4 \equiv -\theta_6.$$

Recall the coefficients in a probit model are identified up to scale (because of the normalization on the standard deviation in probit), provided the support of the explanatory variables has full rank. Thus, in our model the variation in x and x^* on the equilibrium path helps us recover $\beta_j^* \equiv \tilde{\beta}_j/\sigma_\eta$ for $j = 0, 1, 2, 3, 4\}$ (and therefore $(\theta_1 - \theta_4)/\sigma_\eta$, $(\theta_2 - \theta_5)/\sigma_\eta$ and θ_0/σ_η, θ_3/σ_η, θ_6/σ_η) from the conditional probability of incompleteness given x, x^* and job.

Next, note the expectation of the negotiated transfer conditional on $x, x^*, job = 1, d = 1$ is

$$\beta_{1,0} + (\beta_{1,1} + \beta_{1,4})(x^* - x) + \beta_{1,2}x^2 + \beta_{1,3}x^{*2} + \beta_{1,5}x^*x + E[\varepsilon \mid x, x^*, job = 1, d = 1] \quad (10)$$

where by the bivariate normality of (ε, η), the last conditional expectation is the product of a constant coefficient and the inverse mill's ratio evaluated at

$$\beta_0^* + (\beta_1^* + \beta_2^*)(x^* - x) + \beta_3^*(x^{*2} - x^2) + \beta_4^*(x^* - x)^2$$

where $\{\beta_j^* : j = 0, 1, 2, 3, 4\}$ are identified from the selection equation (10). Thus $\beta_{1,0}, \beta_{1,1} + \beta_{1,4}, \beta_{1,2}, \beta_{1,3}, \beta_{1,5}$ are identified using (10) under a typical rank condition.[2]

It then follows that $\theta_6/\theta_3 = -\beta_4^*/\beta_3^* \equiv r^*$ and

$$\begin{aligned}
\frac{\beta_{1,2}}{\beta_{1,3}} &= \frac{(1-\gamma_1)\theta_6 - \gamma_1\theta_3}{(1-\gamma_1)\theta_6 + \gamma_1\theta_3} = \frac{r^*(1-\gamma_1) - \gamma_1}{r^*(1-\gamma_1) + \gamma_1} \\
\Rightarrow \gamma_1 &= \frac{r^*(\beta_{1,3} - \beta_{1,2})}{r^*(\beta_{1,3} - \beta_{1,2}) + \beta_{1,2} + \beta_{1,3}}. \qquad (11)
\end{aligned}$$

[2]The rank condition requires the support of $x^* - x, x^2, x^{*2}, x^*x$ and the mill's ratio to have full rank. This condition can be satisfied under mild conditions due to the nonlinearity of the mill's ratio (even when the selection equation does not involve any exogenous variable that is excluded from the outcome equation). See the last but one paragraph on page 806 in Wooldridge (2010) (which starts with "As a technical point, we do not ...") for more detailed discussion.

Once γ_1 is identified, so is

$$\begin{aligned}\theta_0 &= \beta_{1,0}/(1-\gamma_1);\ \sigma_\eta = -\theta_0/\beta_0^* = \frac{\beta_{1,0}}{(1-\gamma_1)\beta_0^*};\\ \theta_3 &= \sigma_\eta\beta_3^* = \frac{\beta_{1,0}\beta_3^*}{(1-\gamma_1)\beta_0^*};\ \theta_6 = -\sigma_\eta\beta_4^* = -\frac{\beta_{1,0}\beta_4^*}{(1-\gamma_1)\beta_0^*}.\end{aligned}$$

Likewise, we can identify $\beta_{0,0}, \beta_{0,1}, \beta_{0,2}, \beta_{0,3}, \beta_{0,5}$ from the conditional expectation of transfer given $x, x^*, job = 0, d = 1$ which equals

$$\beta_{0,0} + \beta_{0,1}(x^* - x) + \beta_{0,2}x^2 + \beta_{0,3}x^{*2} + \beta_{0,5}x^*x + E[\varepsilon \mid x, x^*, job = 0, d = 1].$$

Then by symmetric argument as in (11), one can identify γ_0 using $\beta_{0,2}, \beta_{0,3}$ in place of $\beta_{1,2}, \beta_{1,3}$.

It remains to show $\theta_1, \theta_2, \theta_4, \theta_5$ are identified. Note that we have already recovered: (a) $\gamma_1\theta_1 + (1-\gamma_1)\theta_4 + [\gamma_1\theta_2 + (1-\gamma_1)\theta_5]$ from the outcome equation conditional on $job = 1$; (b) $\gamma_0\theta_1 + (1-\gamma_0)\theta_4$ from the outcome equation conditional on $job = 0$; and (c) $\theta_1 - \theta_4$ and $\theta_2 - \theta_5$ from the selection equation (10) using σ_η, which is already identified. Thus we can construct a linear system of four equations and four unknowns

$$\begin{pmatrix} \gamma_0 & 1-\gamma_0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ \gamma_1 & 1-\gamma_1 & \gamma_1 & 1-\gamma_1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_4 \\ \theta_2 \\ \theta_5 \end{pmatrix} = r.h.s.$$

where the right-hand side consists of quantities that are already identified. The coefficient matrix on the l.h.s. is non-singular for all $\gamma_0, \gamma_1 \in [0, 1]$. It then follows $\theta_1, \theta_2, \theta_4, \theta_5$ are all identified. Finally, recall that the identification result above was shown by conditioning on w, which means the bargaining power, as a function of characteristics w and the job type, is identified for all values w and job. The identification of ϱ then follows from the parametric form of $\gamma(w, job; \varrho)$ and the full-rankness of the support of w.

References

Matzkin, R. L. (2003): "Nonparametric estimation of nonadditive random functions," *Econometrica*, 71(5), 1339–1375. 1