Uncovering Heterogeneous Social Effects in Binary Choices

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Abstract

We identify and estimate heterogeneous social effects within groups of individuals that make binary choices. These heterogeneous social effects, which include peer and contextual effects, are modeled through *unobserved* influence matrices that summarize how the members within each group affect each other's outcomes. We recover the influence matrices together with other parameters in social effects by exploiting how these matrices are linked to the reduced-form effects of multiple characteristics. Monte Carlo experiments show that a nested fixed-point maximum-likelihood estimator for the social effects has good finite-sample performance. Using a new dataset, we analyze how college roommates influence each other's decisions to participate in volunteering activities. Our estimates reveal substantial heterogeneity in the social effects among these students.

Keywords: heterogeneous peer effects, influence matrix, exclusion restriction, binary choice, volunteering

JEL Classifications: C31; C35; C57; A22

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1 Introduction

In lots of socioeconomic environments, individual decision-makers are partitioned into well-defined groups. Members within a group affect each other's outcomes through the potential influence of their decisions and characteristics (also known as peer and contextual effects respectively). In this paper we address two prominent empirical challenges in the analysis of such models. First, the data available to a researcher does not directly measure the existence or strength of such influence between group members. Second, such influence is generally heterogeneous across paired group members, e.g., it depends on the characteristics of the individuals involved.

We show how to recover heterogeneous *social effects*, which include both *peer effects* and *contextual effects*, from binary choices made by group members. We model these heterogeneous effects through *unknown* influence matrices that vary with individual characteristics and vary across the groups in the data. Accounting for such heterogeneous social effects is important for policy analyses. For example, consider an educator whose goal is to maximize average test scores in multiple classes, and suppose peer effects are known to be positive and greater among students with similar demographics. Then the practice of allocating students with similar demographics into the same class will have a different implication on the class averages than forming classes with greater demographic variety. Quantifying the difference in outcomes between these alternative policies would require a precise measure of the heterogeneous social effects mentioned above.

The identification question we address is empirically motivated: how can a researcher estimate these heterogeneous social effects when the data does not provide any measure of existence or strength of the links between group members? To illustrate, consider a group with n members. Let G denote an n-by-n *influence matrix* whose (i, j)-th component $G_{ij} \in [0, 1]$ is a continuous measure proportional to j's social effects on i; G varies across the groups in the data, is common knowledge within the group, but is not reported in the data observed by researchers. Each individual i's outcome is a binary choice determined as follows:

$$Y_i = 1\Big\{X_i\beta + \sum_{j\leq n} G_{ij}X_j\gamma + \rho\sum_{j\leq n} G_{ij}\mathbb{E}(Y_j|X_1, .., X_n, G) - \varepsilon_i \geq 0\Big\}, \qquad (1)$$

where $1\{\cdot\}$ is a logical operator that returns one when the argument is true and zero otherwise; X_i is a row vector of i's characteristics commonly known within the group (with β and γ being column vectors of parameters); ε_i is a scalar shock known to i only; the conditional expectation of Y_j in the inequality denotes i's equilibrium belief about j's decision (defined later). Then j's social effects on i consist of a *contextual effect* $G_{ij}X_j\gamma$ and a *peer effect* $\rho G_{ij}\mathbb{E}(Y_j|X_1, .., X_n, G)$. Our goal is to recover the parameters (β, γ, ρ) and G.

We treat utility shocks in Equation (1) as private types in a simultaneous game with incomplete information, and characterize the endogenous beliefs under the solution concept of *Bayesian-Nash equilibria*. We impose conditions on the model primitives to guarantee the existence and uniqueness of equilibrium conditional on every possible profile of individual characteristics.

Our identification strategy is novel in that it exploits a key insight: the latent random influence matrix determines the reduced-form effects of all characteristics in the same fashion. The strategy requires an *exclusion restriction* that certain characteristic is known to be excluded from the influence matrix. We take several steps to identify the model elements. First, invert the vector of conditional choice probabilities to obtain latent expected utility indexes. Then, recover the reduced-form coefficients of all characteristics from these indexes. By construction, these reduced-form coefficients are functions of the structural parameters and certain moment of the latent random influence matrix. We recover the structural parameters, utilizing the reduced-form coefficients of multiple regressors and using the exclusion restriction.

Based on this identification argument, we propose a nested fixed-point maximum-likelihood estimator for social effects, and show that it is root-n consistent and asymptotically normally distributed. We investigate its finite sample performance in Monte Carlo experiments. Our simulation exercises illustrate excellent performance of the estimator in terms of average bias and mean-squared error; it roughly converges at a root-n rate given moderate sample sizes of 200, 400, and 800.

We apply our method to study heterogeneous social effects in volunteering decisions by college students, using a new dataset we collected from a university in China. In this setting, the dormitory rooms correspond to the groups in our model. We document substantial differences in social effects across individuals and groups. Our estimates suggest that the social effects are significantly stronger between individuals with similar characteristics. This pattern is consistent with homophily. Our counterfactual analysis shows that allocating similar individuals to the same dormitory room overall leads to more volunteering activities than total random assignments.

In the remainder of this section we discuss related literature. The rest of the paper unfolds as follows. Section 2 introduces a binary choice model with heterogeneous social effects. Section 3 presents a constructive identification method. We propose a nested fixed-point maximum-likelihood estimator for a parameterized model in Section 4 and establish its asymptotic property. We demonstrate the finite sample performance of this estimator through Monte Carlo experiments in Section 5, and conduct an empirical study of college students' volunteering choices in Section 6. Section 7 concludes.

Related Literature

Our work is related to the literature on *social interaction.* Manski (1993) provided non-identification results in social interaction models where individual outcomes are continuous and linear in group means. He showed that peer and contextual effects can not be separated in this model due to a "reflection problem." Graham and Hahn (2005) dealt with this issue using an assumption that some characteristics have no contextual effects. Graham (2008) used second-moment restrictions and variation in group sizes to identify linear-in-means social interaction models. Brock and Durlauf (2001, 2007) established identification in binary choice models with social interaction. None of the specifications in these papers nests ours, which allows heterogeneous social effects and unobserved, stochastic influence matrix.

Our paper is also related to the literature of *social network* models with continuous outcomes. In such models the social effects operate through individual-specific indexes that assign heterogeneous weights to the choices and characteristics of other members. Lee (2007) and Bramoullé, Djebbari, and Fortin (2009) proposed an instrumental-variable approach to disentangle contextual and peer effects, assuming that the researcher observes the influence matrix, a.k.a. the *network structure*. Blume, Brock, Durlauf, and Jayaraman (2015) provided identification results when the researcher knows which pairs of individuals have nonzero influences on each other. Patacchini, Rainone, and Zenou (2017) allowed peer effects to be determined by observed strength of links/friendships, but required knowledge of the network structure.

de Paula, Rasul, and Souza (2019) showed that the network structure, when unreported in the data, can be jointly recovered with other structural parameters from continuous outcomes, provided the data contains many time periods during which the unobserved network structure is fixed. Their method builds on mathematical tools for solving systems of nonlinear equations, and was illustrated in an example with single characteristics. As such, the method does not exploit an intrinsic structural relation between the reduced-form effects of different individual characteristics. Lewbel, Qu, and Tang (2019) took advantage of such a structural relation, and introduced an new identification strategy for cross-sectional data. They treated the unobserved random network structure as nuisances that vary across the groups, and focused on recovering the constant coefficients in social effects. Their identification strategy is constructive, and conducive to a closed-form estimator for social effects based on the analog principle. The method we propose in this paper is related to Lewbel, Qu, and Tang (2019) only in the sense that it also exploits a model-implied relation between the reduced-form effects of multiple individual characteristics. The exact form of such relation, the identification argument, and the estimator proposed are all qualitatively different from theirs. It is worth mentioning that in our case we can also recover the latent influence matrix (network structure) as a parameter of interest along with the social effect parameters.

This paper fills a gap in the literature by disentangling and estimating heterogeneous peer and contextual effects when individuals make binary decisions over network structures (influence matrices) that are not observed in the data. Lin and Xu (2017) studied binary choices on social networks where peer effect parameters are determined by individuals' relative centrality. Lin (2019) introduced quantile-specific peer effect parameters that vary with the level of latent variables underlying observed binary choices. Our model differs substantially from these two papers in that we capture heterogeneous peer effects through latent influence matrices not observed by researchers.

In summary, our method recovers heterogeneous social effects from binary choices when the influence matrix is not reported in the data; these features set the paper apart from the existing literature.

2 The Model

Consider a dataset of individual characteristics and choices collected from many independent groups, each consisting of n members. To reiterate, individuals make simultaneous binary choices under the rule in Equation (1), which is summarized as follows in matrix notation:

$$Y = \mathbf{1}\left\{X\beta + GX\gamma + \rho G\mathbb{E}(Y|X,G) - \varepsilon \geq \mathbf{0}\right\}, \tag{2}$$

where $Y := (Y_i)_{i\leq n}$ is an n-vector of individual binary choices/outcomes; $\mathbf{1}\{\mathbf{v} \geq \mathbf{0}\} := (1\{v_i \geq 0\})_{i\leq n}$ for any n-vector $\mathbf{v}$; $X := (X_1', \cdots, X_n')'$ is an n-by-K matrix of commonly observed characteristics of all members (it does not include a constant column); G is an n-by-n **influence matrix** (with $G_{ii} = 0$ for all i by convention of the literature); $\mathbb{E}(Y_j|X,G) \in [0,1]$ is i's expectation of Y_j conditional on public information within the group; and $\varepsilon := (\varepsilon_i)_{i\leq n}$ is a vector of shocks privately observed by each member. The influence matrix G varies across the groups. For each group, G is known to all members but not reported in the data.

For each $i \leq n$, the vector of individual characteristics is partitioned so that $X_i = (W_i, Z_i)$. Denote $W := (W_1', \cdots, W_n')'$ and $Z := (Z_1', \cdots, Z_n')'$. Correspondingly, we write $X = (W, Z)$, and let $\beta = ((\beta^w)', (\beta^z)')'$ and $\gamma = ((\gamma^w)', (\gamma^z)')'$. We maintain that the influence matrix is a function of the characteristics W while Z is excluded in its determination. Abusing notation, we write $G = G(W)$. This flexible specification allows individual characteristics to determine their social effects on each other. At the same time, it keeps the estimation and interpretation of heterogeneous social effects feasible because the contextual and peer effects $\gamma G(W)$ and $\rho G(W)$ are constant conditional on W. This specification does rule out a more general case of stochastic social effects, e.g., possibly due to unobserved individual heterogeneity in the influence matrix.

We also assume that each ε_i is drawn independently from a known distribution (e.g., standard logistic) with a CDF F and a PDF f that is supported on $(-\infty, \infty)$ and bounded above by $\sup_\varepsilon f(\varepsilon)$; the distribution F does not depend on (X, G).

The solution concept we use for determining Y and individual expectation is pure-strategy **Bayesian-Nash equilibria** of a simultaneous game of incomplete information played within the group. Given the decision rule in Equation (1), we define a **best response function** $BR : [0,1]^n \to [0,1]^n$ so that for any n-vector $\mathbf{p} \in [0,1]^n$,

$$BR_i(\mathbf{p}) = F\Big(X_i\beta + \sum_{j\leq n} G_{ij}X_j\gamma + \rho\sum_{j\leq n} G_{ij}p_j\Big). \tag{3}$$

A fixed point of BR defines an equilibrium of the game. We impose the following regularity condition to ensure equilibrium uniqueness.

Assumption 1. *For any W, the entries of $G(W)$ are nonnegative and bounded from above by a positive constant C, i.e., $\sup_w G(w) < C$. The strength of interaction is moderate so that $|\rho|C \leq \frac{1}{(n-1)\sup_\varepsilon f(\varepsilon)}$.*

Assumption 1 is related to a "moderate social influence" condition of Horst and Scheinkman (2006); the existence and uniqueness of equilibrium in analogous games of incomplete information are proved in Lin and Xu (2017), Xu (2018), Hu and Lin (2019), and Liu (2019). Similarly, under Assumption 1, BR is a contraction mapping (which follows from Blackwell's sufficient conditions), and thus the game admits a unique equilibrium. We omit the proof for lack of novelty.

Proposition 1. *Under Assumption 1, BR defines a contraction mapping; there exists a unique pure-strategy Bayesian-Nash equilibrium in the game of simultaneous binary choices.*

For this fixed point of BR, we write $\mathbf{p}(X) \coloneqq \mathbb{E}(Y|X, G(W)) = \mathbb{E}(Y|X)$, the vector of expected probability in the equilibrium. From Equation (3), we obtain

$$p_i(X) = F\Big(X_i\beta + \sum_{j\leq n} G_{ij}X_j\gamma + \rho\sum_{j\leq n} G_{ij}p_j(X)\Big), \tag{4}$$

where $p_j(X) \coloneqq (\mathbf{p}(X))_j$ for all $j \leq n$. With F known, we invert Equation (4) to get

$$\mathbf{q}(X) \coloneqq (F^{-1}(p_i(X)))_{i\leq n} = \delta(W) + Z\beta^z + G(W)Z\gamma^z + \rho G(W)\mathbf{p}(X), \tag{5}$$

where $\delta(W) \coloneqq W\beta^w + G(W)W\gamma^w$ denotes direct and contextual effects attributable to W. The left-hand side of Equation (5) is identified from the data. In what follows, we suppress W in $G(W)$ and $\delta(W)$ to simplify notation.

3 Identification

We first illustrate our identification strategy using a simple case where $W_i \in \mathbb{R}$ and $Z_i \in \mathbb{R}^2$. Let $\beta^z = (\beta_1^z, \beta_2^z)'$ and $\gamma^z = (\gamma_1^z, \gamma_2^z)'$. Let $\mu^{(1)} \coloneqq \beta_1^z I + \gamma_1^z G$, $\mu^{(2)} \coloneqq \beta_2^z I + \gamma_2^z G$, and $\mu^{(0)} \coloneqq \rho G$ denote n-by-n matrices of reduced-form coefficients.

For any matrix M, denote its i-th row and j-th column respectively by M_{ri} and M_{cj}. Write Equation (5) for each i as

$$F^{-1}(p_i(X)) = \delta_i + \mu_{ri}^{(1)}Z_{c1} + \mu_{ri}^{(2)}Z_{c2} + \mu_{ri}^{(0)}\mathbf{p}(X).$$

Define $V \coloneqq (1, Z'_{c1}, Z'_{c2}, \mathbf{p}(X)')$. We maintain the following rank condition.

Assumption 2. *For any W, $\mathbb{E}(V'V|W)$ is non-singular.*

Under this condition, variation in Z and non-linearity of $\mathbf{p}(\cdot)$ in X help us recover the reduced-form parameters $\mu_{ri}^{(0)}$, $\mu_{ri}^{(1)}$, $\mu_{ri}^{(2)}$, and δ_i from $F^{-1}(p_i(X))$ at any W for each $i \leq n$. Hence, $\mu^{(0)}$, $\mu^{(1)}$, $\mu^{(2)}$, and the vector δ are identified at all values of W. The idea of using nonlinearity in $\mathbf{p}(\cdot)$ to identify binary choices under social interaction was proposed in Brock and Durlauf (2007). In our case such nonlinearity only helps us to recover the reduced-form parameters in this preliminary step. The rank condition we need is analogous to the identifying condition in a special case of the probit selection model in Heckman (1976), where there is no excluded instruments in the auxiliary/selection equation.[1]

Our first step is to identify $(\rho, \beta^z, \gamma^z)$, using a structural relation between $(\mu^{(0)}, \mu^{(1)}, \mu^{(2)})$ revealed in Lemma 1 below. As noted earlier, we suppress the generic argument W in $G(\cdot)$ and $\mu^{(k)}(\cdot)$ to simplify notation.

Lemma 1. *If the 2-by-2 matrix $[\beta^z, \gamma^z]$ is non-singular, then the linear system*

$$a\mu^{(1)} + b\mu^{(2)} = \mu^{(0)} \tag{6}$$

admits a unique solution

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \beta_1^z & \beta_2^z \\ \gamma_1^z & \gamma_2^z \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ \rho \end{pmatrix}.$$

We relegate all proofs to the appendix. For the rest of this section, we assume non-singularity of $[\beta^z, \gamma^z]$ and $\rho \neq 0$.

[1] See Wooldridge (2010, page 806) for details.

Lemma 1 implies two linear restrictions on the parameter vector $(\rho, \beta^z, \gamma^z)$. In addition, it is clear from Equation (2) that a scale normalization is needed for jointly identifying ρ, γ, and G. Thus, we normalize the sum of the first row in $G(\cdot)$ to 1 at a specific value $W = W^*$. For $k = 1$ and 2, let m_k denote the sum of the first row in $\mu^{(k)}$ at W^*, which is equal to $\beta_k^z + \gamma_k^z$. Combine these two equations with the implication of Lemma 1 to get

$$\begin{pmatrix} 0 & a & b & 0 & 0 \\ -1 & 0 & 0 & a & b \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \rho \\ \beta_1^z \\ \beta_2^z \\ \gamma_1^z \\ \gamma_2^z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ m_1 \\ m_2 \end{pmatrix}.$$

The rank of the coefficient matrix is full because $a = b = 0$ is ruled out by non-singular $[\beta^z, \gamma^z]$ and nonzero ρ.

Given these four linear restrictions, an additional restriction is needed for point identification of $(\rho, \beta^z, \gamma^z)$. We maintain that $\gamma_2^z = 0$, that is, the second characteristic in Z has no contextual effect. Such **exclusion restriction** has been used in the literature for identifying social interaction models, e.g., in Graham and Hahn (2005). The choice of excluded characteristics depends on institutional details in the specific empirical contexts considered.

Next, with ρ identified, we recover $G(\cdot)$ from $\mu^{(0)}(\cdot)$ at all values of W. Then, with $\delta(\cdot)$ and $G(\cdot)$ known for all W, we identify β^w and γ^w from $\delta(\cdot)$ under a mild support condition that is congruent with nonlinearity of $G(\cdot)$.

Assumption 3. *The support of* $[W_{ri}, G_{ri}(W)W]$ *is not included in a proper linear subspace of* $\mathbb{R}^2$ *at least for some* $i \leq n$.

We summarize the identification results in the following theorem, whose proof is already presented in the text above.

Theorem 1. *Given Assumptions 1, 2, and 3, the parameters ρ, β, γ, and $G(\cdot)$ are identified.*

It is straightforward to generalize this identification strategy to the cases with higher-dimensional $W_i \in \mathbb{R}^{K_w}$ and $Z_i \in \mathbb{R}^{K_z}$. In such cases, Lemma 1 holds for $\mu^{(k)}$ and $\mu^{(K_z)}$ for $k = 1, 2, ..., K_z - 1$ with $a_k, b_k \in \mathbb{R}$ such that

$$\begin{pmatrix} \beta_k^z & \beta_{K_z}^z \\ \gamma_k^z & \gamma_{K_z}^z \end{pmatrix} \begin{pmatrix} a_k \\ b_k \end{pmatrix} = \begin{pmatrix} 0 \\ \rho \end{pmatrix}.$$

One can construct $2(K_z - 1) + K_z$ equations in $(\rho, \beta^z, \gamma^z) \in \mathbb{R}^{2K_z+1}$:

$$\begin{pmatrix} \mathbf{0}_{(K_z-1)\times 1} & A & \mathbf{0}_{(K_z-1)\times K_z} \\ -\mathbf{1}_{(K_z-1)\times 1} & \mathbf{0}_{(K_z-1)\times K_z} & A \\ \mathbf{0}_{K_z\times 1} & I & I \end{pmatrix} \begin{pmatrix} \rho \\ \beta^z \\ \gamma^z \end{pmatrix} = \begin{pmatrix} \mathbf{0}_{(K_z-1)\times 1} \\ \mathbf{0}_{(K_z-1)\times 1} \\ \mathbf{m} \end{pmatrix},$$

where I is a $K_z \times K_z$ identity matrix; $A := [diag(a_1, ..., a_{K_z-1}),\ (b_1, b_2, ..., b_{K_z-1})']$ is a (K_z-1)-by-K_z matrix; and $\mathbf{m} := (m_1, m_2, ..., m_{K_z})'$ with m_k being the sum of the first row in $\mu^{(k)}$ at the fixed value W^* for which the scale normalization is introduced. By construction, the coefficient matrix has rank $2K_z$ generically. With an additional exclusion restriction such as $\gamma_k^z = 0$ for a known k, the augmented linear system has full rank to point identify ρ, β^z, and γ^z.

Remark 1. Lewbel, Qu, and Tang (2019) established a result similar to Lemma 1 in a linear social network model where the dependent variable (outcome) is continuous and the influence matrix G is latent (not reported in data) and stochastic across the groups. They regressed individual outcomes on the characteristics of group members to obtain reduced-form coefficients, which depend on certain moment of the latent random G and the other structural coefficients in social effects. Their goal is to identify and infer these structural coefficients while leaving the distribution of G as an unspecified nuisance parameter. In contrast, the dependent variable in our model is binary, and the latent influence matrix, after controlling for W, is fixed and treated as a parameter of interest to be recovered together with the structural coefficients.

Remark 2. It is worth noting that we can extend our method to identify a more general model where $\rho(\cdot)$, $\beta^z(\cdot)$, and $\gamma^z(\cdot)$ are unknown functions of W. In this case, one need to normalize the sum of the first row in $G(W)$ to 1 for each W. This is without loss of generality, because the scale of $\rho(W)$ and $G(W)$ cannot be pinned down for each W in Equation (2).

To identify this model, first recover the reduced-form coefficients $\mu^{(k)}(\cdot)$, $\mu^{(0)}(\cdot)$, and $\delta(\cdot)$ conditional on each W as above. Then, apply Lemma 1 and the subsequent argument conditional on W to identify $\beta^z(\cdot)$ and $\gamma^z(\cdot)$ for each W, maintaining appropriate rank and exclusion restrictions. Lastly, with $\rho(W)$ identified, recover $G(W) = \mu^{(0)}(W)/\rho(W)$ for each W.

Alternatively, one can identify $\rho(W)$ as the sum of the first row in $\mu^{(0)}(W)$ for each W. In the next step, the linear system derived from Lemma 1 and the normalization would only treat $\beta^z(\cdot)$ and $\gamma^z(\cdot)$ as unknown parameters. One still needs an additional restriction to point identify β^z and γ^z because the rank of the coefficient matrix of these $2K_z$ parameters is $2K_z - 1$ generically.

4 Nested Fixed Point Estimation

We define a **nested fixed-point maximum likelihood estimator** (Rust, 1987) in a model where dependence of G on W is parametrized for the sake of tractability. Suppose the sample contains S groups, each with n members. Abusing notation, we let $Y_s := (Y_{s1}, \cdots, Y_{sn})'$, $X_s := (X'_{s1}, \cdots, X'_{sn})'$, $W_s := (W'_{s1}, \cdots, W'_{sn})'$, and $\varepsilon_s := (\varepsilon_{s1}, \cdots, \varepsilon_{sn})'$. Let $G_s = G(W_s; \lambda)$, where λ is a finite-dimensional parameter fixed across groups. We make the following parametric assumption on utility shocks.

Assumption 4. *The random utility shocks ε_{si} are independent across individuals and groups, and follow the standard Logistic distribution.*

Let $\theta := (\rho, \beta', \gamma', \lambda)$ denote the vector of all structural parameters, and θ_0 denote the true parameter. We also need the following condition.

Assumption 5. *θ_0 is in the interior of a compact parameter space Θ; the support of X_s is bounded. For all possible λ and ρ, $G(W; \lambda)$ is nonnegative and bounded from above by a constant $C > 0$ with $\sup_w G(w; \lambda) < C$ and $|\rho| C \leq \frac{1}{(n-1)\sup_\varepsilon f(\varepsilon)}$.*

The first two conditions in this assumption are standard for asymptotic theory. Under the third condition, the model admits a unique equilibrium for all values of λ and ρ in the parameter space.

Write $\sigma^*_s(\theta) := (\sigma^*_{s1}(\theta), \cdots, \sigma^*_{sn}(\theta))' = \mathbb{E}(Y_s | X_s; \theta)$, which is well-defined and can be calculated as a fixed point in Equation (4) for each X_s and θ given knowledge of shock distribution and unique equilibrium. The log-likelihood function is

$$\hat{L}_S(\theta) \coloneqq \frac{1}{S}\sum_{s=1}^{S} \hat{l}_s(\theta). \tag{7}$$

where $\hat{l}_s(\theta) \coloneqq \sum_{i=1}^{n}[Y_{si} \cdot \log \sigma^*_{si}(\theta) + (1 - Y_{si}) \cdot \log(1 - \sigma^*_{si}(\theta))]$. Let $\hat{\theta} = \underset{\theta \in \Theta}{\operatorname{argmax}}\, \hat{L}_S(\theta)$ be the nested fixed-point maximum likelihood estimator. Define $L_0(\theta) \coloneqq \mathbb{E}[\hat{l}_1(\theta)]$

Assumption 6. *The true parameter θ_0 uniquely maximizes $L_0(\theta)$.*

Assumption 7. $\Omega(\theta_0) \coloneqq \mathbb{E}\left[\frac{\partial \hat{l}_1(\theta_0)}{\partial \theta} \times \frac{\partial \hat{l}_1(\theta_0)}{\partial \theta'}\right]$ *exists and is nonsingular.*

Assumption 6 is the identification condition needed for consistency of maximum likelihood estimators. Section 3 showed nonparametric identification of ρ, β, γ, and $G(W)$ under Assumptions 1 to 3. Thus Assumption 6 holds if in addition $G(\cdot;\lambda) \neq G(\cdot;\tilde{\lambda})$ for all $\tilde{\lambda} \neq \lambda$. Assumption 7 is a typical rank condition needed for deriving the limit distribution of the estimator.

Under these maintained assumptions, a nested fixed-point maximum-likelihood estimator is root-n consistent and asymptotically normally distributed.

Theorem 2. *Under Assumptions 1 to 6, $\hat{\theta} \xrightarrow{p} \theta_0$. If in addition Assumption 7 holds,*

$$\sqrt{S}(\hat{\theta} - \theta_0) \xrightarrow{d} N\big(0, \Omega(\theta_0)^{-1}\big).$$

5 Monte Carlo Experiments

This section investigates the finite sample performance of the estimator in Section 4 by Monte Carlo experiments. Fix $n = 4$, which is a typical size of groups in our empirical application in Section 6 (college dormitory rooms). The vector of individual characteristics is partitioned into $W_i = (W_{i1}, W_{i2})$ and $Z_i = (Z_{i1}, Z_{i2})$, with the latter excluded in the influence matrix $G(W)$. Let (W_{i1}, W_{i2}, Z_{i1}) be mutually independent and follow the standard normal distribution; let Z_{i2} be Bernoulli with expectation $\frac{1}{2}$. The latter is intended to mimic empirical situations where certain instrument has a small, discrete support.

Each individual makes a binary decision following the unique equilibrium strategy described in our model. Let $\Lambda(t) \coloneqq \frac{e^t}{1+e^t}$ denote the CDF of the standard logistic distribution, and $\lambda \coloneqq (\lambda_1, \lambda_2)$ be parameters in the social influence matrix G. For individuals $i \neq j$, $G_{ij}(W) = \Lambda(\lambda_1 |W_{i1} - W_{j1}| + \lambda_2 |W_{i2} - W_{j2}|)$. (Recall that $G_{ii}(W) = 0$ by convention.) As noted earlier, we normalize the sum of the first row in G to 1.

Random utility shocks ε_i are drawn independently from the standard logistic distribution. Our experiments follow a 3-by-3 factorial design: we consider different sample sizes $S = 200$, 400, and 800, and different true peer effect parameters $\rho_0 = 1$, 2, and 4, while keeping other parameters constant with $(\beta_0, \gamma_0, \lambda_0) = (1, -1, 1, -1, 1, -1, 1, -1, 1, -1)$. For each experimental condition, we draw individual characteristics and random utility shocks, calculate influence matrices and equilibria, and estimate parameters using the

Table 1: Monte Carlo Experiments

$\rho_0 = 1;\ \beta_0 = (1,-1,1,-1);\ \gamma_0 = (1,0,1,-1); \lambda_0 = (-1,-1)$										
S	ρ	β				γ_1	γ_3	γ_4	λ	
	Average Bias									
200	-0.007	0.020	-0.023	0.022	-0.014	0.052	0.071	-0.040	-0.045	-0.020
400	0.020	0.008	-0.008	0.010	-0.010	0.029	0.033	-0.029	-0.020	-0.051
800	0.005	0.006	-0.008	0.006	-0.012	0.010	0.013	0.009	-0.015	0.005
	Mean Squared Error									
200	0.316	0.015	0.037	0.021	0.059	0.120	0.210	0.226	0.181	0.263
400	0.122	0.006	0.018	0.009	0.028	0.051	0.090	0.104	0.059	0.119
800	0.060	0.004	0.008	0.005	0.013	0.026	0.044	0.048	0.030	0.051

$\rho_0 = 2;\ \beta_0 = (1,-1,1,-1);\ \gamma_0 = (1,0,1,-1); \lambda_0 = (-1,-1)$										
S	ρ	β				γ_1	γ_3	γ_4	λ	
	Average Bias									
200	0.051	0.021	-0.032	0.025	-0.018	0.042	0.057	-0.029	-0.015	-0.022
400	0.036	0.010	-0.008	0.008	-0.013	0.030	0.030	-0.024	-0.017	-0.030
800	0.019	0.007	-0.009	0.003	-0.010	0.013	0.017	0.004	-0.016	0.005
	Mean Squared Error									
200	0.218	0.014	0.033	0.020	0.058	0.092	0.160	0.182	0.076	0.147
400	0.096	0.006	0.017	0.009	0.030	0.044	0.071	0.092	0.035	0.074
800	0.049	0.004	0.007	0.005	0.013	0.020	0.035	0.042	0.019	0.032

$\rho_0 = 4;\ \beta_0 = (1,-1,1,-1);\ \gamma_0 = (1,0,1,-1); \lambda_0 = (-1,-1)$										
S	ρ	β				γ_1	γ_3	γ_4	λ	
	Average Bias									
200	0.131	0.028	-0.033	0.021	-0.035	0.061	0.069	-0.020	-0.0160	-0.015
400	0.077	0.007	-0.010	0.006	-0.021	0.037	0.039	0.000	-0.014	-0.014
800	0.078	0.008	-0.011	0.002	-0.015	0.022	0.034	0.005	-0.024	-0.011
	Mean Squared Error									
200	0.241	0.016	0.027	0.019	0.060	0.067	0.103	0.162	0.028	0.045
400	0.115	0.007	0.013	0.009	0.030	0.028	0.051	0.072	0.013	0.025
800	0.065	0.004	0.006	0.004	0.013	0.017	0.023	0.035	0.007	0.012

nested fixed point method; the process is replicated for 1000 times so that we can report the average bias and mean squared errors in Table 1.

In all experimental designs, our estimators recover true parameters quite well. There is evidence that the mean squared errors roughly follow root-n convergence as sample sizes increase.

6 Peer Effects in Volunteering Activities

In this section we apply our method to investigate the peer effects in college students' decisions to participate in volunteering activities, using a new dataset collected from a university in China.

6.1 Data Description

The dataset we use includes administrative records of demographic information, dormitory room assignment, and volunteering activities of all freshmen students enrolled at the university in 2015. These students also participated in a survey as the class of 2019 graduates in a related survey (AEA RCT Registry number AEARCTR-0004296).[2] In this cohort, a total of 3,982 freshmen were assigned to 955 dormitories, each of which accommodated 3, 4, or 6 students. University documents showed that the administrators in charge of room assignment were asked to maximize student diversity in each room in terms of majors of study, provinces of origin, and ethnic groups.[3] The university did not admit any international undergraduate students. Among all students in the cohort, 3,569 stayed in the same room throughout four years of college.

Our sample consists of 1,964 students assigned to 491 four-person dorms.

[2] The public URL for the trial is: http://www.socialscienceregistry.org/trials/4296

[3] Among these students, 3,781 were ethnic Hans, the ethnic majority in China. Therefore no two minority students were assigned to the same room.

Interviews with students and university administrators suggested that these roommates typically took different classes (because they had different majors) and rarely interacted outside the dormitory.

At the beginning of academic year 2016-17, the university started to use a mobile phone application to keep track of student participation in certain on-campus and off-campus activities. The goal was to collect input data for calculating a "comprehensive performance index" for each student. The index was important for students seeking university scholarships, because scholarship eligibility requires a minimal level of the comprehensive performance index as well as GPA. The index is calculated on a full scale of 200 points: 100 for GPA, and 100 for a measure of "quality development." In an academic year, the students could obtain 5 quality development points for participation in every 20 hours of volunteering activities. These volunteering activities ranged from helping childless elders to assisting the organization of academic conferences.

We keep track of this cohort for two academic years between 2016 and 2018. In 2016-17, 77.2% of these 1,964 students participated in at least one volunteering activity (as measured by a dummy covariate *PrevYrVol*); the percentage dropped to 46.9% due to busier schedules (according to our outcome/dependent variable *Volunteer*): most students started preparing for admission exams for domestic or international graduate schools, or job markets. We include four additional covariates: *Age* (measured by birth year and month), *Rural* (whether the student was registered as a member of an "agricultural family" in China's nationwide household registration system), *SingleChild* (whether the student had no siblings), and *Science* (whether the student took a science-based or liberal arts-based format of the national college entrance examination). Table 2 reports the summary statistics.

Table 2: Summary Statistics for the Student Sample

Variable	Mean	Standard Deviation
Age	18.533	0.650
Science	0.512	0.500
Rural	0.321	0.467
SingleChild	0.704	0.457
PrevYrVol	0.772	0.419
Volunteer	0.469	0.499

6.2 Estimation and Counterfactual Analysis

We estimate the social effects in students' binary decisions to participate in volunteering activity during the 2017-2018 academic year. The social influence matrix G is parametrized as in Section 4. We include covariates (*Science*, *Rural*, *SingleChild*) in W_i, and (*Age*, *PrevYrVol*) in Z_i excluded from the social influence matrix. Our choice of variables in Z_i is based on a focus group study of students. We also maintain a working assumption that *Age* has no immediate contextual effect, i.e., $\gamma_{age} = 0$. This is plausible because the roommates in our sample were from the same cohort and it is unlikely that small differences in birth months would have non-trivial exogenous effects.

Table 3 reports the nested fixed point maximum likelihood estimates with standard errors. The estimates for direct effect coefficients (β) for *Age*, *Rural*, *SingleChild*, and *PrevYrVol* are all statistically significant with negative, positive, negative, and positive signs respectively. This suggests that, absent contextual and social effects, a younger student with a rural origin, no siblings, and some volunteering experience in the previous year would be more inclined to volunteer than students with other demographic features.

The estimates for contextual effect coefficients (γ) for *Science* and *SingleChild* are both statistically significant with negative signs; the contextual effect for *PrevYrVol* is statistically positive. These signs are all consistent with those of direct individual effects in β. In comparison, having a roommate with a rural origin appears to reduce the likelihood to volunteer, even though the direct effect of *Rural* on one's own volunteering decision is positive. The results indicate that, with more roommates from science, rural, and single-child backgrounds, an individual student tends to volunteer less. These results do not contradict our institutional knowledge.

Our estimate of the peer effect coefficient $\hat{\rho} = 0.739$ is statistically significant, confirming some source of simultaneity and mutual influence between volunteering decisions by peers/roommates.

Table 3 also reports how W_i affects the influence matrix through the coefficients in λ. The estimated coefficients for (*Science*, *Rural*, *SingleChild*) are $\hat{\lambda} = (-1.078, -1.062, -0.680)$, all of which are statistically significant. These estimates provide evidence for homophily among the students. That is, roommates with similar demographic features tend to have stronger social influence on each other. Moreover, these estimates also demonstrate strong heterogeneity in social effects between students, which would have been lost if we had assumed that the students only influenced each other in a symmetric, homogeneous fashion (e.g., if G were specified as a row-normalization of binary 0 or 1 components that indicate the existence of links).

Table 3: Estimation for Volunteering Choices

	Estimate	Standard Errors
Direct Effects (β)		
Age	-0.092**	0.004
Science	-0.050	0.041
Rural	0.299**	0.035
SingleChild	-0.176**	0.026
PrevYrVol	1.718**	0.037
Contextual Effects (γ)		
Science	-0.435**	0.072
Rural	-0.336**	0.098
SingleChild	-0.195**	0.064
PrevYrVol	0.517**	0.189
Influence Matrix Parameters (λ)		
Science	-1.078**	0.235
Rural	-1.062**	0.444
SingleChild	-0.680**	0.232
Peer Effects (ρ)	0.739**	0.334

* and **: 10% and 5% significant.

To highlight the degree of heterogeneity in social effects, we plot the density of the average of off-diagonal components in the social influence matrix G across the dormitories in Figure 1. The bell-shaped pattern in the density suggests that rooms with extremely low or high average social influences are rare. Heterogeneity of social effects is also present within dormitories: Figure 2 plots the density of standard deviations between off-diagonal components of G across all dorms. This density demonstrates a bell shape too.

Our structural approach is of particular interest for policymakers, because it can be used to analyze how different schemes of roommate assignments would affect student participation in volunteering activities. To illustrate, we conduct counterfactual exercises which calculate the numbers of predicted volunteering

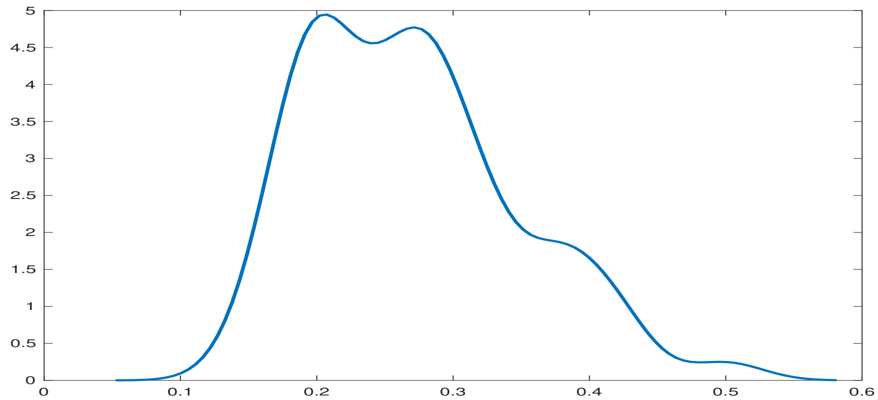

Figure 1: Density of Mean Influence Across Groups

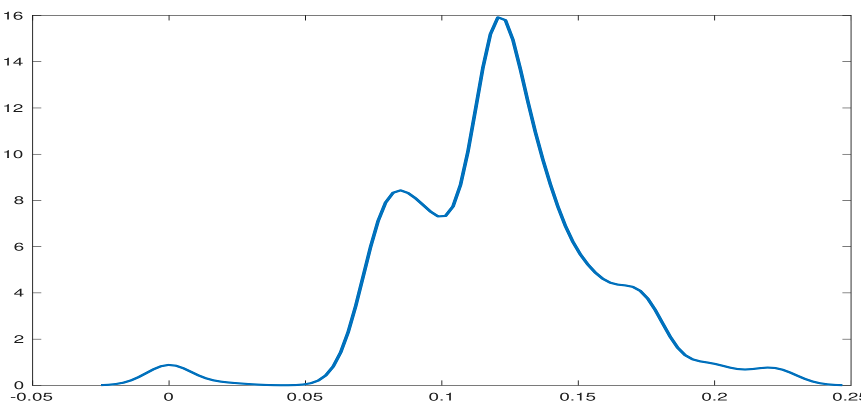

Figure 2: Density of Standard Deviation of Influence Across Groups

activities when students are assigned to rooms under different schemes. We follow a 2-by-3 factorial design. Based on three different covariates *Science*, *Rural*, and *SingleChild*, we either form maximally mixed dormitory rooms or maximally segregated ones, from the given pool of students. For example, 51.2% of our sample took a science-based format of the national college entrance examination. We generate two counterfactual datasets: in the first dataset, each room has two science students and two non-science students; in the second dataset, half of the rooms (245) only have science students and the other half only have non-science students. The distribution of all other covariates are the same as in the real sample. We draw utility shocks, and calculate the numbers of volunteering students in the two datasets. Table 4 reports the results for all designs. Mixing leads to marginally fewer incidences of volunteering.

Table 4: Counterfactual Analysis for Volunteering Choices

Volunteering out of 1,964 Students		
	Mixing	Segregating
Science	955	1008
Rural	972	1002
SingleChild	957	987

7 Conclusions

We identify and estimate heterogeneous social effects in scenarios where modeling these effects as dichotomous is inadequate, and the extent to which a person's choice is influenced by another (or whether it is influenced at all) is unobserved to the researcher.

Considering a dataset with many independent groups of individuals, we propose a binary choice model which centers on latent influence matrices that vary across the groups. These matrices are determined by individual characteristics and proportional to the social effects. We recover the influence matrices together with other parameters in social effects by exploiting how these matrices are linked to the reduced-form effects of multiple characteristics. We estimate the model with a nested fixed-point estimator, and illustrate its finite-sample performance through Monte Carlo experiments. Based on data obtained from a university, an empirical study of social effects in college students' volunteering activities indicates substantial heterogeneity in these effects as well as the presence of homophily in social influence.

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Appendix A Proofs

A.1 Proof of Lemma 1

Recall that $\mu^{(k)} := \beta_k^z I + \gamma_k^z G$. By construction, $a\beta_1^z + b\beta_2^z = 0$ and $a\gamma_1^z + b\gamma_2^z = \rho$. Thus, the left-hand side of Equation (6) equals $0 + \rho G := \mu^{(0)}$.

To see the uniqueness of the solution, consider $(\tilde{a}, \tilde{b}) \neq (a, b)$, and denote $c := \tilde{a}\beta_1^z + \tilde{b}\beta_2^z$ and $d := \tilde{a}\gamma_1^z + \tilde{b}\gamma_2^z$. Because (β^z, γ^z) has full rank, $(c, d) \neq (0, \rho)$. Next, note that the diagonal entries in G and the off-diagonal entries in I are zeros. Hence,

$$\tilde{a}\mu^{(1)} + \tilde{b}\mu^{(2)} = cI + dG \neq \rho G$$

and Equation (6) does not hold for such $(\tilde{a}, \tilde{b})$. □

A.2 Proof of Theorem 2

The proof is similar to that of Theorem 2.1 in Newey and McFadden (1994).

By our identification result, $L_0(c)$ is uniquely maximized at θ_0. By the definition of MLE, for any $\epsilon > 0$, we have $\hat{L}_S(\hat{\theta}) > \hat{L}_S(\theta_0) - \frac{1}{3}\epsilon$ with probability approaching one (w.p.a.1). By the law of large number, we have that $L_0(\hat{\theta}) > \hat{L}_S(\hat{\theta}) - \frac{1}{3}\epsilon$ and $\hat{L}_S(\theta_0) > L_0(\theta_0) - \frac{1}{3}\epsilon$.

Therefore, w.p.a.1,

$$L_0(\hat{\theta}) > \hat{L}_S(\hat{\theta}) - \frac{1}{3}\epsilon > \hat{L}_S(\theta_0) - \frac{2}{3}\epsilon > L_0(\theta_0) - \epsilon. \tag{8}$$

Let $\mathcal{N}$ be any open subset of Θ containing θ_0. Since $\Theta \cap \mathcal{N}^c$ is compact, θ_0 uniquely maximizes $L_0(\theta)$, and $L_0(\theta)$ is continuous, we have

$$\sup_{\theta \in \Theta \cap \mathcal{N}^c} L_0(\theta) = L_0(\theta^*) < L_0(\theta_0).$$

Therefore, for $\epsilon = L_0(\theta_0) - \sup_{\theta \in \Theta \cap \mathcal{N}^c} L_0(\theta)$, we have w.p.a.1,

$$L_0(\hat{\theta}) > \sup_{\theta \in \Theta \cap \mathcal{N}^c} L_0(\theta).$$

Hence, $\hat{\theta} \in \mathcal{N}$.

By the definition of $\hat{\theta}$, we have $\frac{\partial \hat{L}(\hat{\theta})}{\partial \theta} = 0$. Taylor expansion gives us

$$\frac{\partial \hat{L}_S(\theta_0)}{\partial \theta} + \frac{\partial^2 \hat{L}_S(\tilde{\theta})}{\partial \theta \partial \theta'}(\hat{\theta} - \theta_0) = 0,$$

where $\tilde{\theta}$ is between $\hat{\theta}$ and θ_0. Since we have independent repeated small blocks $s = 1, \cdots, S$, a standard central limit theorem applies to $\frac{\partial \hat{L}_S(\theta_0)}{\partial \theta}$; by a classical information equality, we have

$$\sqrt{S}(\hat{\theta} - \theta_0) \xrightarrow{d} N(0, \Omega(\theta_0)^{-1}).$$

□