Uncovering Heterogeneous Social Effects in Binary Choices *

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Abstract

We identify and estimate heterogeneous social effects within groups of individuals that make binary choices. These heterogeneous social effects, which include peer and contextual effects, are modeled through *unobserved* influence matrices that summarize how the members within each group affect each other's outcomes. We recover parameters in social effects as well as the unknown influence matrices by exploiting how these matrices are linked to the reduced-form effects of multiple characteristics. Monte Carlo experiments show that a nested fixed-point maximum-likelihood estimator for the social effects has good finite-sample performance. Using a new dataset, we analyze how college roommates influence each other's decisions to participate in volunteering activities. Our estimates reveal substantial heterogeneity in the social effects among these students.

Keywords: heterogeneous peer effects, influence matrix, exclusion restriction, binary choice, volunteering

JEL Classifications: C31; C35; C57; A22

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1 Introduction

In lots of socioeconomic environments, individual decision-makers are partitioned into well-defined groups. Members within a group affect each other's outcomes through the potential influence of their decisions and characteristics (also known as peer and contextual effects respectively). In this paper we address two prominent empirical challenges in the analysis of such models. First, the data available to a researcher does not directly measure the existence or strength of such influence between group members. Second, such influence is generally heterogeneous across paired group members, e.g., it depends on the characteristics of the individuals involved.

We show how to recover heterogeneous *social effects*, which include both *peer effects* and *contextual effects*, from binary choices made by group members. We model these heterogeneous effects through *unknown* influence matrices that vary with individual characteristics and vary across the groups in the data. Accounting for such heterogeneous social effects is important for policy analyses. For example, consider an educator whose goal is to maximize average test scores in multiple classes, and suppose peer effects are known to be positive and greater among students with similar demographics. Then the practice of allocating students with similar demographics into the same class will have a different implication on the class averages than forming classes with greater demographic variety. Quantifying the difference in outcomes between these alternative policies would require a precise measure of the heterogeneous social effects mentioned above.

The identification question we address is empirically motivated: how can a researcher estimate these heterogeneous social effects when the data does not provide any measure of existence or strength of the links between group members? To illustrate, consider a group with n members. Let G denote an n-by-n *influence matrix* whose (i, j)-th component $G_{ij} \in [0, 1]$ is a continuous measure proportional to j's social effects on i; G varies across the groups in the data, is common knowledge within the group, but is not reported in the data observed by researchers. Naturally, G_{ij} may derive from the strength of friendship; it can also incorporate other information such as the amount of time spent together in the past. Each individual i's outcome is a binary choice determined as follows:

$$Y_i = 1\Big\{X_i\beta + \sum_{j\leq n} G_{ij}X_j\gamma + \rho\sum_{j\leq n} G_{ij}\mathbb{E}(Y_j|X_1, .., X_n, G) - \varepsilon_i \geq 0\Big\}, \qquad (1)$$

where $1\{\cdot\}$ is a logical operator that returns one when the argument is true and zero otherwise; X_i is a row vector of i's characteristics commonly known within the group (with β and γ being column vectors of parameters); ε_i is a scalar shock known to i only; the conditional expectation of Y_j in the inequality denotes i's equilibrium belief about j's decision (defined later). Then j's social effects on i consist of a *contextual effect* $G_{ij}X_j\gamma$ and a *peer effect* $\rho G_{ij}\mathbb{E}(Y_j|X_1, .., X_n, G)$. Our goal is to recover the parameters (β, γ, ρ) and G.

We treat utility shocks in Equation (1) as private types in a simultaneous game with incomplete information, and characterize the endogenous beliefs under the solution concept of *Bayesian-Nash equilibria*. We impose conditions on the model primitives to guarantee the existence and uniqueness of equilibrium conditional on individual characteristics.

Our identification strategy is original in that it exploits a key insight: the latent influence matrix determines the reduced-form effects of all characteristics in the same fashion. The strategy requires an *exclusion restriction* that certain characteristics are known to be excluded from the influence matrix. We take several steps to identify the model elements. First, invert the vector of conditional choice probabilities to obtain latent expected utility indexes, and recover the reduced-form coefficients of all characteristics from these indexes. By construction, these reduced-form coefficients are functions of the structural parameters and certain moment of the latent random influence matrix. Next, we recover the structural parameters utilizing a linear relation between the reduced-form coefficients for different regressors implied by the model structure. This relation allows us to construct a linear system for identifying the structural parameters, aided with exclusion restrictions such as no contextual effect for a known characteristic.

Based on this identification argument, we propose a nested fixed-point maximum-likelihood estimator for social effects, and show that it is root-n consistent and asymptotically normally distributed. We investigate its finite sample performance in Monte Carlo experiments. Our simulation exercises illustrate excellent performance of the estimator in terms of average bias and mean-squared error; it converges at a root-n rate given moderate sample sizes of 200, 400, and 800.

We apply our method to study heterogeneous social effects in volunteering decisions by college students, using a new dataset we collected from a university in China. In this setting, the dormitory rooms correspond to the groups in our model. We document substantial differences in social effects across individuals and groups. Our estimates suggest that the social effects are significantly stronger between individuals with similar characteristics. This pattern is consistent with homophily. Our counterfactual analysis shows that allocating similar individuals to the same dormitory room overall leads to more volunteering activities than total random assignments.

The rest of the paper unfolds as follows. In Section 2, we discuss related literature. Section 3 introduces a binary choice model with heterogeneous social effects. Section 4 presents a constructive identification method. We propose a nested fixed-point maximum-likelihood estimator for a parameterized model in Section 5 and establish its asymptotic property. We demonstrate the finite sample performance of this estimator through Monte Carlo experiments in Section 6, and conduct an empirical study of college students' volunteering choices in Section 7. Section 8 concludes.

2 Related Literature

Our paper is related to the literature on *social interaction*. Manski (1993) provided non-identification results in social interaction models where individual outcomes are continuous and linear in group means. He showed that peer and contextual effects cannot be separated in this model due to a "reflection problem." Graham and Hahn (2005) dealt with this issue using an assumption that some characteristics have no contextual effects. Graham

(2008) used second-moment restrictions and variation in group sizes to identify linear-in-means social interaction models. Brock and Durlauf (2001, 2007) established identification for binary choices in social interaction models where each group member's decision is influenced by a rational expectation of the average choice in the group; and Lee, Li, and Lin (2014) achieved identification and provided maximum likelihood estimation for a general network model with heterogenous rational expectations. None of the specifications in these papers nests ours, which allows heterogeneous social effects and unobserved influence matrix.

Bajari, Hong, Krainer, and Nekipelov (2010) estimated static games of incomplete information with multiple equilibria. Florens and Sbaï (2010) provided general criteria for local identification in games of incomplete information. Aradillas-Lopez (2010, 2012) estimated semiparametric games of incomplete information. De Paula and Tang (2012) proposed a test for multiple equilibria and signs of interaction effects in static games with incomplete information. Wan and Xu (2014) showed identification in semiparametric games of incomplete information with correlated private signals. In comparison, we study social networks with simultaneous binary choices and private information, where the latent network structure leads to heterogeneous social effects. Our identification strategy is original and exploits structural links between the reduced-form partial effects and the latent social effects. Such links are distinctive features of the social network models we study.

Since the seminal work of Bresnahan and Reiss (1991), there has been a growing literature on empirical games with *complete information*. A challenge in such models is to deal with the model incompleteness and identification under multiple equilibria. Tamer (2003) estimated binary games with complete information, building on robust implications under multiple equilibria. Krauth (2006) and Soetevent and Kooreman (2007) proposed simulated maximum likelihood estimation for complete-information models with peer effects. Li and Zhao (2016) estimated discrete games with complete information on large networks. They adopted a partial identification approach and proposed feasible estimation, which introduced a novel way to use the inequalities implied by lower-dimensional subnetworks.

Our paper is also related to the literature of social networks with *continuous* outcomes. In such models, the social effects operate through individual-specific indexes that assign heterogeneous weights to other members' choices and characteristics. Lee (2007) and Bramoullé, Djebbari, and Fortin (2009) used instruments to disentangle contextual and peer effects, assuming researchers observe the *network structure*, i.e., the influence matrices of our model. Blume et al. (2015) provided identification results when the researcher knows which pairs of individuals have nonzero influences on each other. Patacchini, Rainone, and Zenou (2017) allowed peer effects to be determined by observed strength of links/friendships, but required knowledge of the network structure. Boucher and Houndetoungan (2020) estimated peer effects in social networks when researchers know, or already have a consistently estimate of, the distribution of network structure. They provided an original method to use this information to estimate peer effects in a linear-in-means model.

De Paula, Rasul, and Souza (2019) showed that the network structure, when unreported in the data, can be jointly recovered with other structural parameters from continuous outcomes, provided the data contains many time periods with a fixed network structure. Their method builds upon mathematical tools for solving systems of nonlinear equations, and does not exploit an intrinsic structural relation between the reduced-form effects of different individual characteristics. Lewbel, Qu, and Tang (2019) took advantage of such a structural relation, and introduced a new identification strategy that only requires cross-sectional data. They treated the unobserved network structures as nuisances varying across the groups, and focused on recovering the constant coefficients in social effects. Their identification strategy is constructive, and conducive to a closed-form estimator for social effects. The method we propose in this paper is related to Lewbel, Qu, and Tang (2019) only in the sense that it also exploits a model-implied relation between the reduced-form effects of multiple individual characteristics. The exact form of such relation, the identification argument, and the estimator proposed in our paper are all qualitatively different from theirs. It is worth mentioning that in our case we can also recover the latent influence matrix (network structure) as a parameter of interest along with the other social effect parameters.

This paper fills a gap in the literature by disentangling and estimating heterogeneous peer and contextual effects when individuals make binary decisions over unobserved influence matrices. Lin and Xu (2017) studied binary choices on social networks where peer effect parameters are determined by individuals' relative centrality. Lin (2019) introduced quantile-specific peer effect parameters that vary with the level of latent variables underlying observed binary choices. Our model differs substantially from these two papers in that we capture heterogeneous peer effects through latent influence matrices unobserved by researchers.

3 The Model

Consider a dataset of individual characteristics and choices collected from many independent groups, each consisting of n members. To reiterate, individuals make simultaneous binary choices as in Equation (1), which is summarized in matrix notation as:

$$Y = \mathbf{1}\left\{X\beta + GX\gamma + \rho G\mathbb{E}(Y|X,G) - \varepsilon \geq \mathbf{0}\right\}, \tag{2}$$

where $Y \coloneqq (Y_i)_{i\leq n}$ is an n-vector of individual binary choices/outcomes; $\mathbf{1}\{\mathbf{v} \geq \mathbf{0}\} \coloneqq (1\{v_i \geq 0\})_{i\leq n}$ for any n-vector $\mathbf{v}$; $X \coloneqq (X_1', \cdots, X_n')'$ is an n-by-K matrix of commonly observed characteristics of all members (it does not include a constant column); G is an n-by-n **influence matrix**, with $G_{ii} = 0$ for all i by convention of the literature and $G_{ij} \in [0,1]$ for all i and j; $\mathbb{E}(Y_j|X,G) \in [0,1]$ is i's expectation of Y_j conditional on public information within the group; and $\varepsilon \coloneqq (\varepsilon_i)_{i\leq n}$ is a vector of shocks privately observed by each member. Parameters β and γ are K-by-1, and ρ is a scalar. We assume $\rho \neq 0$ to rule out triviality. The influence matrix G varies across the groups. For each group, G is known to all members but not reported in the data.

For each $i \leq n$, the vector of individual characteristics is partitioned so that $X_i = (W_i, Z_i)$. Denote $W := (W_1', \cdots, W_n')'$ and $Z := (Z_1', \cdots, Z_n')'$. Correspondingly, we write $X = (W, Z)$, and let $\beta = ((\beta^w)', (\beta^z)')'$ and $\gamma = ((\gamma^w)', (\gamma^z)')'$. We maintain that the influence matrix is a function of W but not Z. Abusing notation, we write $G = G(W)$. This flexible specification allows individual characteristics to determine their social effects on each other. At the same time, it keeps the estimation and interpretation of heterogeneous social effects feasible because the contextual and peer effects $\gamma G(W)$ and $\rho G(W)$ are constant conditional on W. This specification does rule out a more general case of stochastic social effects, e.g., possibly due to unobserved individual heterogeneity in the influence matrix.

We also assume that each ε_i is drawn independently from a known distribution (e.g., standard logistic) with a CDF F and a PDF f that is supported on $(-\infty, \infty)$ and bounded above by $\sup_\varepsilon f(\varepsilon) < \infty$; the distribution F does not depend on (X, G).

The solution concept we use for determining Y and individual expectation is pure-strategy **Bayesian-Nash equilibria** of a simultaneous game of incomplete information played within each group. Given the decision rule in Equation (1), we define a **best response function** $BR : [0,1]^n \to [0,1]^n$ so that for any n-vector $\mathbf{p} = (p_1, \cdots, p_n)' \in [0,1]^n$,

$$BR_i(\mathbf{p}) = F\Big(X_i\beta + \sum_{j \le n} G_{ij} X_j \gamma + \rho \sum_{j \le n} G_{ij} p_j\Big). \tag{3}$$

A fixed-point of BR defines an equilibrium of the game.

Assumption 1. *The sample is generated from a single Bayesian-Nash equilibrium conditional on X.*

Under this assumption, the equilibrium beliefs of group members are directly identified and can be consistently estimated as the conditional choice probabilities (CCPs) given X from the sample. Without this assumption, the estimator of CCPs from the sample would converge in probability to a mixture of different CCPs from multiple equilibria, and would not satisfy a fixed-point characterization in Equation (4) below.

Similar assumptions of single equilibrium in the sample are used in other papers such as Pesendorfer and Schmidt-Dengler (2008), Bajari, Hong, Krainer, and Nekipelov (2010), and Lewbel and Tang (2015). In principle, the assumption of single equilibrium can be tested using the information from the sample. De Paula and Tang (2012, 2020) provided tests for multiple equilibria in simultaneous Bayesian games with independent and correlated private signals respectively.

It is worth mentioning that Assumption 1 is implied by a mild condition of moderate social influence (Glaeser and Scheinkman, 2000; Horst and Scheinkman, 2006).

Assumption (MSI). *There exists a constant* $C > 0$ *such that* $\sup_w G(w) < C$ *and* $|\rho| C \leq \frac{1}{(n-1)\sup_\varepsilon f(\varepsilon)}$.

This condition restricts the modulus, or strength of interaction in the model, so that the best response function is a contraction mapping. Under Assumption (MSI), the model only admits a unique equilibrium. Therefore, it implies Assumption 1.[1] Lee, Li, and Lin (2014), Lin and Xu (2017), Xu

[1]This modulus condition is not directly testable unless one maintains Assumption 1. In that case, one can estimate the model under Assumption 1 and then use the estimate and

(2018), Hu and Lin (2019), and Liu (2019) used similar conditions to show uniqueness of equilibrium in Bayesian games.

We write the fixed-point of BR as $\mathbf{p}(X) \coloneqq \mathbb{E}(Y|X, G(W)) = \mathbb{E}(Y|X)$, which is a column vector of expected probability in the equilibrium. From Equation (3), we obtain

$$p_i(X) = F\Big(X_i\beta + \sum_{j\leq n} G_{ij}X_j\gamma + \rho\sum_{j\leq n} G_{ij}p_j(X)\Big), \tag{4}$$

where $p_j(X)$ is the j-th component of $\mathbf{p}(X)$. Invert Equation (4) to get

$$\mathbf{q}(X) \coloneqq (F^{-1}(p_i(X)))_{i\leq n} = \delta(W) + Z\beta^z + G(W)Z\gamma^z + \rho G(W)\mathbf{p}(X), \tag{5}$$

where $\delta(W) \coloneqq W\beta^w + G(W)W\gamma^w$ denotes direct and contextual effects attributable to W. With knowledge of F, the left-hand side of Equation (5) is identified from the data. In what follows, we suppress W in $G(W)$ and $\delta(W)$ to simplify notation.

4 Identification

We first illustrate our identification strategy using a simple case where $W_i \in \mathbb{R}$ and $Z_i \in \mathbb{R}^2$. Let $\beta^z = (\beta_1^z, \beta_2^z)'$ and $\gamma^z = (\gamma_1^z, \gamma_2^z)'$. Let $\mu^{(1)} \coloneqq \beta_1^z I + \gamma_1^z G$, $\mu^{(2)} \coloneqq \beta_2^z I + \gamma_2^z G$, and $\mu^{(0)} \coloneqq \rho G$ denote n-by-n matrices of reduced-form coefficients.

For any matrix M, denote its i-th row and j-th column respectively by M_{ri} and M_{cj}. Write Equation (5) for each i as

$$F^{-1}(p_i(X)) = \delta_i + \mu_{ri}^{(1)}Z_{c1} + \mu_{ri}^{(2)}Z_{c2} + \mu_{ri}^{(0)}\mathbf{p}(X).$$

(standard error) for ρ to verify the modulus condition. In our application, we follow these steps to test and verify the modulus condition.

Define $V \coloneqq (1, Z_{c1}', Z_{c2}', \mathbf{p}(X)')$. We maintain the following rank condition.

Assumption 2. $\mathbb{E}(V'V|W)$ *is non-singular for all* W*.*

Assumption 2 is a regularity condition that rules out pathological cases where the equilibrium choice probability $\mathbf{p}(X)$ is linearly dependent with the covariates in Z. Recall that $\mathbf{p}(X)$ solves a nonlinear fixed-point equation in Equation (4). Hence $\mathbf{p}(X)$ is generally nonlinear in Z conditional on W. Bajari, Hong, Krainer, and Nekipelov (2010, Theorem 1) and Aguirregabiria and Mira (2019, Proposition 3) both used similar rank conditions on equilibrium choice probabilities for identification.

Under Assumption 2, variation in Z and non-linearity of $\mathbf{p}(\cdot)$ in X help us to recover the reduced-form parameters $\mu_{ri}^{(0)}$, $\mu_{ri}^{(1)}$, $\mu_{ri}^{(2)}$, and δ_i from $F^{-1}(p_i(X))$ at any W for each $i \leq n$. Hence, $\mu^{(0)}$, $\mu^{(1)}$, $\mu^{(2)}$, and δ are identified at all values of W. The idea of using nonlinearity in $\mathbf{p}(\cdot)$ to identify binary choices under social interaction was introduced in Brock and Durlauf (2007). In our case, such nonlinearity helps us to recover the reduced-form parameters in this preliminary step. Our rank condition is analogous to the identifying condition in the probit selection model in Heckman (1976), where there is no excluded instruments in the auxiliary/selection equation.[2]

Our first step is to identify ρ, β^z, and γ^z, using a structural relation between $\mu^{(0)}$, $\mu^{(1)}$, and $\mu^{(2)}$ revealed in Lemma 1 below. As noted earlier, we suppress the argument W in $G(\cdot)$ and $\mu^{(k)}(\cdot)$ to simplify notation.

[2]See Wooldridge (2010, page 806) for details.

Lemma 1. *If the 2-by-2 matrix* (β^z, γ^z) *is non-singular, then the linear system*

$$a\mu^{(1)} + b\mu^{(2)} = \mu^{(0)} \tag{6}$$

admits a unique solution

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \beta_1^z & \beta_2^z \\ \gamma_1^z & \gamma_2^z \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ \rho \end{pmatrix}.$$

The lemma uses a key insight that the reduced-form effects of both characteristics $\mu^{(1)}$ and $\mu^{(2)}$ depend on the *same* influence matrix G and respective structural coefficients ρ, β_k, and γ_k in the same way. Thus one can exploit this structure to recover a linear relation between these reduced-form effects. Lemma 1 establishes that the weights in this linear relation are a known function of structural coefficients. We relegate proofs to the appendix. For the rest of this section, we assume non-singularity of (β^z, γ^z).

Lemma 1 implies two linear restrictions on the vector $(\rho, \beta^z, \gamma^z)$. In addition, it is clear from Equation (2) that a scale normalization is needed for joint identification of ρ, γ, and G. Thus, we normalize the sum of the first row in $G(\cdot)$ at a specific value $W = W^*$ to one. For $k = 1, 2$, let m_k denote the sum of the first row in $\mu^{(k)}$ at W^*, which is equal to $\beta_k^z + \gamma_k^z$. Combine these two equations with the implication of Lemma 1 to get

$$\begin{pmatrix} 0 & a & b & 0 & 0 \\ -1 & 0 & 0 & a & b \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \rho \\ \beta_1^z \\ \beta_2^z \\ \gamma_1^z \\ \gamma_2^z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ m_1 \\ m_2 \end{pmatrix}.$$

The rank of the coefficient matrix is full because $a = b = 0$ is ruled out by non-singular (β^z, γ^z) and nonzero ρ.

Given these four linear restrictions, an additional restriction is needed for point identification of ρ, β^z, and γ^z. We maintain that $\gamma_2^z = 0$, that is, the second characteristic in Z has no contextual effect. Such **exclusion restriction** has been used in the literature for identifying linear-in-means models of social interactions, e.g., in Graham and Hahn (2005). The choice of excluded characteristics depends on institutional details in the specific empirical contexts considered.

Next, with ρ identified, we recover $G(\cdot)$ from $\mu^{(0)}(\cdot)$ at all values of W. Then, with $\delta(\cdot)$ and $G(\cdot)$ known for all W, we identify β^w and γ^w from $\delta(\cdot)$ under a mild support condition that is congruent with nonlinearity of $G(\cdot)$.

Assumption 3. *The support of $[W_{ri}, G_{ri}(W)W]$ is not included in a proper linear subspace of $\mathbb{R}^2$ at least for some $i \leq n$.*

We summarize the identification results in the following theorem, whose proof is already presented in the text above.

Theorem 1. *If the exclusion restriction ($\gamma_2^z = 0$) and Assumptions 1, 2, and 3 hold, then ρ, β, γ, and $G(\cdot)$ are identified.*

It is straightforward to generalize this identification strategy to higher dimensions with $W_i \in \mathbb{R}^{K_w}$ and $Z_i \in \mathbb{R}^{K_z}$ for $K_z \geq 3$ and $K_w \geq 2$. In such cases, Lemma 1 holds for $\{\mu^{(k)}\}_{k=1,2,\ldots,K_z-1}$ and $\mu^{(K_z)}$ with $a_k, b_k \in \mathbb{R}$

given by

$$\begin{pmatrix} \beta_k^z & \beta_{K_z}^z \\ \gamma_k^z & \gamma_{K_z}^z \end{pmatrix} \begin{pmatrix} a_k \\ b_k \end{pmatrix} = \begin{pmatrix} 0 \\ \rho \end{pmatrix}.$$

One can construct $2(K_z - 1) + K_z$ equations in $(\rho, \beta^z, \gamma^z) \in \mathbb{R}^{2K_z+1}$:

$$\begin{pmatrix} \mathbf{0}_{(K_z-1)\times 1} & A & \mathbf{0}_{(K_z-1)\times K_z} \\ -\mathbf{1}_{(K_z-1)\times 1} & \mathbf{0}_{(K_z-1)\times K_z} & A \\ \mathbf{0}_{K_z\times 1} & I & I \end{pmatrix} \begin{pmatrix} \rho \\ \beta^z \\ \gamma^z \end{pmatrix} = \begin{pmatrix} \mathbf{0}_{(K_z-1)\times 1} \\ \mathbf{0}_{(K_z-1)\times 1} \\ \mathbf{m} \end{pmatrix},$$

where I is a $K_z \times K_z$ identity matrix; $A := [diag(a_1, ..., a_{K_z-1}),\ (b_1, b_2, ..., b_{K_z-1})']$ is a (K_z-1)-by-K_z matrix; and $\mathbf{m} := (m_1, m_2, ..., m_{K_z})'$ with m_k being the sum of the first row in $\mu^{(k)}$ at the fixed value W^* by which the scale normalization is introduced. By construction, the coefficient matrix has rank $2K_z$ generically. With an additional exclusion restriction such as $\gamma_k^z = 0$ for a known k, the augmented linear system has full rank to point identify the structural coefficients ρ, β^z, and γ^z.

With such exclusion restrictions and $K_z \geq 3$, the system generally over-identifies the structural coefficients. This is because there exist multiple ways for taking $2K_z+1$ out of the $2(K_z-1)+K_z+1$ equality restrictions that have sufficient ranks for point identification. One may also append the linear system with further equations constructed from any linear constraints or additional exclusion restrictions that are acceptable in the context. (See Section 5.4 in Lewbel, Qu, and Tang (2019) for details about the use of additional exclusion restrictions.) This over-identifying power can be exploited for improving estimation efficiency in a likelihood-based or two-step GMM estimation.

Remark 1. Lewbel, Qu, and Tang (2019) established a result similar to Lemma 1 in linear social networks where the dependent variable (outcome) is *continuous* and the influence matrices G are latent (not reported in data) and vary across the groups. They regressed individual outcomes on the characteristics of group members to obtain reduced-form coefficients, which depend on some moments of the latent G and the structural coefficients in social effects. Their goal is to identify and infer these structural coefficients while leaving the distribution of G as a nuisance parameter. In contrast, the dependent variable in our model is binary, and the latent influence matrix, after controlling for W, is fixed and treated as a parameter of interest to be recovered together with the structural coefficients.

Remark 2. It is worth noting that we can extend our method to identify a more general model where $\rho(\cdot)$, $\beta^z(\cdot)$, and $\gamma^z(\cdot)$ are unknown functions of W with $\rho(W) \neq 0$ for all W. In this case, one needs to normalize the sum of the first row in $G(W)$ to 1 for each W. This is without loss of generality, because the scale of $\rho(W)$ and $G(W)$ cannot be pinned down for each W in Equation (2).

To identify this model, we first recover the reduced-form coefficients $\mu^{(k)}(\cdot)$, $\mu^{(0)}(\cdot)$, and $\delta(\cdot)$ conditional on each W as above. Then, apply Lemma 1 and the subsequent argument conditional on W to identify $\rho(\cdot)$, $\beta^z(\cdot)$, and $\gamma^z(\cdot)$ for each W, maintaining appropriate rank and exclusion restrictions. Lastly, with $\rho(\cdot)$ identified, we recover $G(W) = \mu^{(0)}(W)/\rho(W)$ for each W.

Alternatively, one can identify $\rho(W)$ as the sum of the first row in $\mu^{(0)}(W)$ for each W. In the next step, the linear system derived from Lemma 1 and the normalization would only treat $\beta^z(W)$ and $\gamma^z(W)$ as unknown parameters. One still needs an additional restriction to point identify $\beta^z(W)$ and $\gamma^z(W)$ because the rank of the coefficient matrix of these $2K_z$ parameters is $2K_z - 1$ generically.

Remark 3. If the private shocks $(\varepsilon_i)_{i \leq n}$ are correlated across group members conditional on X, then each member i's expectation of others' choices $\mathbb{E}[Y_j | X, G, \varepsilon_i]$ must also depend on i's own shock non-trivially (because Y_j is a function of ε_j, which is correlated with ε_i conditional on X). In this case, the definition of Bayesian-Nash equilibrium is different, and our identification method does not apply even with full parametrization of the joint distribution of shocks.

When private shocks are correlated, a pure strategy of a group member i is defined by $\mathcal{S}_i(x)$, which is a subset of the support of private shock ε_i given $X = x$, such that i chooses 1 if and only if $\varepsilon_i \in \mathcal{S}_i(x)$. In equilibrium, member beliefs about others are consistent with the choice probabilities implied by these sets $\{\mathcal{S}_i(x)\}_{i \leq n}$. With additional assumptions on model primitives, one can establish the existence of monotone pure-strategy equilibria, where each member i follows a threshold-crossing strategy to choose 1 whenever ε_i crosses a threshold $t_i(x)$.[3] In matrix notation, the vector of equilibrium thresholds $(t_i(X))_{i \leq n}$ in this case solves the fixed-point problem for each X:

$$X_i \beta + G_i X_i \gamma + \rho G_i \mathbb{E}[1\{\varepsilon \leq t\} | X, G, \varepsilon_i = t_i] = t_i$$

[3] A sufficient condition for existence of monotone pure strategy Bayesian-Nash equilibria is that the interim payoff for each member satisfies the single-crossing condition defined in Athey (2001).

for all $i \leq n$, where X_i and G_i denote the i-th rows in X and G respectively, and $1\{\varepsilon \leq t\}$ is shorthand for the n-vector $(1\{\varepsilon_i \leq t_i\})_{i \leq n}$.

Suppose we know that the sample is generated from a single monotone pure-strategy equilibrium, and that the joint distribution of $(\varepsilon_i)_{i \leq n}$ is from a parametric family with non-zero correlation; and normalize its marginals to a known distribution. Then we can invert the CCPs conditional on X to recover $t_i(X)$ for all i, but the interim beliefs $\mathbb{E}[Y|X, G, \varepsilon_i = t_i(X)]$ is not directly identifiable from the data-generating process, because we cannot conditional on private shocks in ε_i when estimating the CCPs. This means we cannot recover the reduced-form coefficients $\{\mu^{(k)}\}_{k=0,1,2}$ in the first step by inverting the CCPs, because the structural link in Equation (4) would now involve the unidentified expectations $\mathbb{E}[Y_j|X, G, \varepsilon_i = t_i(X)]$. With full parametrization, one can write down the likelihood, which requires solving a nested fixed-point problem for each trial parameter value. Nevertheless, even if we adopt such a parametric approach, global identification would require a different approach than the one we use in this paper.

Remark 4. Our model assumes peer and contextual effects share the same influence matrix. This specification is commonly used in the literature of social networks (see Lee, 2007; Bramoullé, Djebbari, and Fortin, 2009; De Paula, Rasul, and Souza, 2019, for example). We capitalize on this specification in Lemma 1 to derive a linear system of equations for identifying the structural coefficients.

One can generalize our method to allow for different influence matrices in peer and contextual effects. That is, the binary outcomes are now given by

$$Y = \mathbf{1}\left\{X\beta + \tilde{G}X\gamma + \rho G\mathbb{E}(Y|X, G, \tilde{G}) - \varepsilon \geq \mathbf{0}\right\}, \tag{7}$$

where G and $\tilde{G}$ are distinct influence matrices for peer and contextual effects respectively. Characterization of equilibrium is similar to Equation (4), except that the contextual influence matrix in front of $X\gamma$ is replaced by $\tilde{G}$. As before, we maintain that influence matrices are functions of W but not Z, and suppress W in notation for simplicity.

We illustrate the identification of this generalized model using a case where $W_i \in \mathbb{R}$, $Z_i \in \mathbb{R}^3$, $\beta^z := (\beta_1^z, \beta_2^z, \beta_3^z)'$, and $\gamma^z := (\gamma_1^z, \gamma_2^z, \gamma_3^z)'$, with $(\beta_3^z, \gamma_3^z) \neq (0, 0)$. Let $\mu^{(k)} := \beta_k^z I + \gamma_k^z \tilde{G}$ for $k = 1, 2, 3$; and let $\mu^{(0)} := \rho G$ denote the n-by-n matrices of reduced-form coefficients.

First, under the same rank condition in Assumption 2, one can recover the reduced-form parameters $\{\mu^{(k)}\}_{k=0,1,2,3}$ by inverting the CCPs $\mathbf{p}(X)$ and using the variation of V conditional on W at any W. With a scale normalization that the first row in G sums up to one at a reference value W^*, we identify ρ as the sum of the first row in μ^0 at W^*.

Next, under the condition that $((\beta_1^z, \beta_2^z)', (\gamma_1^z, \gamma_2^z)')$ has full rank, we can use the same argument as in the proof of Lemma 1 to show that the equation

$$a\mu^{(1)} + b\mu^{(2)} = \mu^{(3)} \tag{8}$$

admits a unique solution

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \beta_1^z & \beta_2^z \\ \gamma_1^z & \gamma_2^z \end{pmatrix}^{-1} \begin{pmatrix} \beta_3^z \\ \gamma_3^z \end{pmatrix}.$$

Equation (8) contrasts with Lemma 1, which establishes a similar linear relation for $\mu^{(1)}$, $\mu^{(2)}$, and $\mu^{(0)}$ when peer and contextual effects are based on the same latent influence matrix. In Equation (8), the linear relation only involves reduced-form coefficients of characteristics in X, which all depend on $\tilde{G}$ alone. The other coefficients in $\mu^{(0)}$ are used earlier for recovering the peer coefficient ρ and the influence matrix G. Equation (8) allows us to construct a linear system of five equations in β^z and γ^z

$$\begin{pmatrix} a & b & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & b & -1 \\ & \mathbf{I} & & & \mathbf{I} & \end{pmatrix} \begin{pmatrix} \beta^z \\ \gamma^z \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{m} \end{pmatrix}, \tag{9}$$

where $\mathbf{I}$ is a 3-by-3 identity matrix, $\mathbf{0}$ is a 2-by-1 vector of zeros, and $\mathbf{m}$ is a 3-by-1 vector $\mathbf{m} := (m_1, m_2, m_3)'$ with m_k being the sum of the first row in $\mu^{(k)}$ at W^*. By construction, the rank of the coefficient matrix in this system of five equations is four generically. To see this, note that the sum of the first two rows in the coefficient matrix in Equation (9) is a linear combination of the last three rows. More generally, with $Z_i \in \mathbb{R}^K$, we can recover a linear relation for $\{\mu^{(k)}\}_{k=0,1,...,K}$ similar to Equation (8), and use it to construct a system of $J := 2(K-2)+K$ equations for $2K$ unknown parameters in β^z and γ^z, similar to Equation (9).[4] Because of the linear dependence noted above, the rank of coefficient matrix in such a system is $J - (K-2) = 2K-2$ generically. For point identification, one can append additional exclusion restrictions to the linear system to increase the rank to $2K$. For example, $\gamma_k = 0$ for a pair of

[4]The number of equations is $2(K-2)+K$ because the analog to Lemma 1 would only imply $K-2$ free restrictions on β^z and γ^z respectively, and there are K restrictions from the sum of the first row in $\mu^{(k)}$ for $k = 1, 2, ..., K$.

known covariates, or exploit exogenous variation in group sizes that does not affect some of the structural coefficients. See Lewbel, Qu, and Tang (2019) for detailed discussion about the types of additional exclusion restrictions that can be used to estimate structural coefficients from such linear systems.

5 Nested Fixed Point Estimation

We define a **nested fixed-point maximum likelihood estimator** (Rust, 1987) in a model where dependence of G on W is parametrized for tractability. Suppose the sample contains S groups, each with n members. Abusing notation, we let $Y_s := (Y_{s1}, \cdots, Y_{sn})'$, $X_s := (X'_{s1}, \cdots, X'_{sn})'$, $W_s := (W'_{s1}, \cdots, W'_{sn})'$, and $\varepsilon_s := (\varepsilon_{s1}, \cdots, \varepsilon_{sn})'$, where the subscript s index groups. Let $G_s = G(W_s; \lambda)$, where λ is a finite-dimensional parameter fixed across groups. We make the following parametric assumption on private shocks.

Assumption 4. *The private shocks ε_{si}'s are independent across individuals and groups, and follow the standard Logistic distribution.*

Let $\theta := (\rho, \beta', \gamma', \lambda)$ denote the vector of all structural parameters, and θ_0 denote the true parameter that generates the sample. We also need the following condition.

Assumption 5. *θ_0 is in the interior of a compact parameter space Θ; the support of X_s is bounded. For all λ, $G(W; \lambda)$ is nonnegative and bounded from above by a constant $C > 0$ such that $\sup_w G(w; \lambda) < C$ and $|\rho| C \leq \frac{1}{(n-1)\sup_\varepsilon f(\varepsilon)}$ for all ρ.*

The first two conditions in this assumption are standard for asymptotic theory. Under the third condition, the model admits a unique equilibrium for all values of λ and ρ in the parameter space.

Write $\sigma_s^*(\theta) \coloneqq (\sigma_{s1}^*(\theta), \cdots, \sigma_{sn}^*(\theta))' = \mathbb{E}(Y_s|X_s;\theta)$, which is well-defined and can be calculated as a fixed point in Equation (4) for each X_s and θ given knowledge of the shock distribution and uniqueness of equilibrium. The log-likelihood function is

$$\hat{L}_S(\theta) \coloneqq \frac{1}{S}\sum_{s=1}^{S} l_s(\theta). \tag{10}$$

where $l_s(\theta) \coloneqq \frac{1}{n}\sum_{i=1}^{n}[Y_{si}\log\sigma_{si}^*(\theta) + (1 - Y_{si})\log(1 - \sigma_{si}^*(\theta))]$. Let $\hat{\theta} = \underset{\theta\in\Theta}{\operatorname{argmax}}\ \hat{L}_S(\theta)$ be the nested fixed-point maximum likelihood estimator. Denote the probability limit of the objective function as $L_0(\theta) \coloneqq \mathbb{E}[l_s(\theta)]$.

Assumption 6. *θ_0 uniquely maximizes $L_0(\theta)$.*

Assumption 7. $\Omega(\theta_0) \coloneqq \mathbb{E}\left[\frac{\partial l_s(\theta_0)}{\partial\theta'} \times \frac{\partial l_s(\theta_0)}{\partial\theta}\right]$ *exists and is non-singular.*

Assumption 6 summarizes the identification condition. Section 4 already provided primitive conditions (Assumptions 1 to 3) for the nonparametric identification of ρ, β, γ, and $G(W)$. Thus Assumption 6 holds if in addition $G(\cdot;\lambda) \neq G(\cdot;\tilde{\lambda})$ for all $\tilde{\lambda} \neq \lambda$. Assumption 7 is a typical rank condition needed for deriving the limit distribution of the estimator.

Under these assumptions, a nested fixed-point maximum-likelihood estimator is root-n consistent and asymptotically normally distributed.

Theorem 2. *Under Assumptions 4 to 6, we have $\hat{\theta} \xrightarrow{p} \theta_0$. If in addition Assumption 7 holds,*

$$\sqrt{S}(\hat{\theta} - \theta_0) \xrightarrow{d} N\big(0, \Omega(\theta_0)^{-1}\big).$$

In some contexts, the groups in the sample have different sizes. As far as identification is concerned, this does not pose any challenge, because the arguments in Section 4 remain valid conditional on any fixed group size n. Furthermore, group size variation is accommodated in our estimation. Specifically, if the structural coefficients do not vary with group sizes, the likelihood in Equation (10) simply sums over groups with different sizes. If the structural coefficients do vary with group sizes, then the likelihood for each group depends on size-specific coefficients, and we can estimate all size-specific coefficients jointly using the nested fixed point MLE, as long as the sample pools sufficiently many observations for each group size.

6 Monte Carlo Experiments

This section demonstrates the finite sample performance of the estimator in Section 5 through Monte Carlo experiments.

Consistent with our model setup, the vector of individual characteristics $X_i = (W_i, Z_i)$ is partitioned into $W_i = (W_{i1}, W_{i2})$ and $Z_i = (Z_{i1}, Z_{i2})$, with the latter excluded from the influence matrix $G(W)$. Let (W_{i1}, W_{i2}, Z_{i1}) be mutually independent and follow the standard normal distribution; let Z_{i2} be Bernoulli with expectation $\frac{1}{2}$. The latter is intended to mimic empirical situations where instruments have a small, discrete support.

Each individual makes a binary decision following an equilibrium strategy in our model. Let $\Lambda(t) := \frac{e^t}{1+e^t}$ denote the CDF of the standard logistic distribution, and $\lambda := (\lambda_1, \lambda_2)$ be parameters in the social influence matrix

Table 1: Monte Carlo Experiments for Group Size 4

S	ρ	β				γ_1	γ_3	γ_4	λ	
$\rho_0 = 1;\ \beta_0 = (1, -1, 1, -1);\ \gamma_0 = (1, 0, 1, -1); \lambda_0 = (-1, -1)$										
	Average Bias									
200	-0.007	0.020	-0.023	0.022	-0.014	0.052	0.071	-0.040	-0.045	-0.020
400	0.020	0.008	-0.008	0.010	-0.010	0.029	0.033	-0.029	-0.020	-0.051
800	0.005	0.006	-0.008	0.006	-0.012	0.010	0.013	0.009	-0.015	0.005
	Mean Squared Error									
200	0.316	0.015	0.037	0.021	0.059	0.120	0.210	0.226	0.181	0.263
400	0.122	0.006	0.018	0.009	0.028	0.051	0.090	0.104	0.059	0.119
800	0.060	0.004	0.008	0.005	0.013	0.026	0.044	0.048	0.030	0.051
$\rho_0 = 2;\ \beta_0 = (1, -1, 1, -1);\ \gamma_0 = (1, 0, 1, -1); \lambda_0 = (-1, -1)$										
S	ρ	β				γ_1	γ_3	γ_4	λ	
	Average Bias									
200	0.051	0.021	-0.032	0.025	-0.018	0.042	0.057	-0.029	-0.015	-0.022
400	0.036	0.010	-0.008	0.008	-0.013	0.030	0.030	-0.024	-0.017	-0.030
800	0.019	0.007	-0.009	0.003	-0.010	0.013	0.017	0.004	-0.016	0.005
	Mean Squared Error									
200	0.218	0.014	0.033	0.020	0.058	0.092	0.160	0.182	0.076	0.147
400	0.096	0.006	0.017	0.009	0.030	0.044	0.071	0.092	0.035	0.074
800	0.049	0.004	0.007	0.005	0.013	0.020	0.035	0.042	0.019	0.032
$\rho_0 = 4;\ \beta_0 = (1, -1, 1, -1);\ \gamma_0 = (1, 0, 1, -1); \lambda_0 = (-1, -1)$										
S	ρ	β				γ_1	γ_3	γ_4	λ	
	Average Bias									
200	0.131	0.028	-0.033	0.021	-0.035	0.061	0.069	-0.020	-0.0160	-0.015
400	0.077	0.007	-0.010	0.006	-0.021	0.037	0.039	0.000	-0.014	-0.014
800	0.078	0.008	-0.011	0.002	-0.015	0.022	0.034	0.005	-0.024	-0.011
	Mean Squared Error									
200	0.241	0.016	0.027	0.019	0.060	0.067	0.103	0.162	0.028	0.045
400	0.115	0.007	0.013	0.009	0.030	0.028	0.051	0.072	0.013	0.025
800	0.065	0.004	0.006	0.004	0.013	0.017	0.023	0.035	0.007	0.012

G. For $i \neq j$, $G_{ij}(W) = \Lambda(\lambda_1 |W_{i1} - W_{j1}| + \lambda_2 |W_{i2} - W_{j2}|)$.[5] As noted earlier,

[5] Recall that $G_{ii}(W) = 0$ by convention. While our identification and estimation approach accommodate asymmetric influence matrices, the Monte Carlo design here and the empirical application in the next section focus on the case with symmetric influence matrices. In these cases, any off-diagonal G_{ij} is better thought of as originating from symmetric components of social interactions such as the strength of friendship and the amount of time spent together,

Table 2: Monte Carlo Experiments for Group Size 6

S	ρ	β				γ_1	γ_3	γ_4	λ	
$\rho_0 = 1;\ \beta_0 = (1, -1, 1, -1);\ \gamma_0 = (1, 0, 1, -1); \lambda_0 = (-1, -1)$										
	Average Bias									
200	0.016	0.016	-0.012	0.016	-0.016	0.025	0.029	-0.028	-0.017	-0.025
400	0.005	0.005	-0.009	0.010	0.005	0.016	0.011	-0.019	-0.008	-0.011
800	0.005	0.004	-0.001	0.004	-0.003	0.004	0.010	-0.011	-0.003	-0.005
	Mean Squared Error									
200	0.081	0.009	0.025	0.016	0.047	0.042	0.067	0.081	0.049	0.079
400	0.033	0.005	0.012	0.008	0.025	0.021	0.032	0.038	0.022	0.041
800	0.017	0.002	0.006	0.004	0.012	0.009	0.016	0.018	0.010	0.019
$\rho_0 = 2;\ \beta_0 = (1, -1, 1, -1);\ \gamma_0 = (1, 0, 1, -1); \lambda_0 = (-1, -1)$										
S	ρ	β				γ_1	γ_3	γ_4	λ	
	Average Bias									
200	0.040	0.014	-0.018	0.008	-0.023	0.026	0.043	-0.014	-0.015	-0.013
400	0.022	0.006	-0.012	0.007	-0.004	0.021	0.013	-0.016	-0.009	-0.004
800	0.010	0.004	-0.002	0.003	-0.008	0.007	0.014	-0.008	-0.006	0.001
	Mean Squared Error									
200	0.059	0.009	0.021	0.015	0.042	0.031	0.053	0.056	0.024	0.043
400	0.030	0.005	0.011	0.007	0.025	0.016	0.024	0.030	0.012	0.024
800	0.014	0.002	0.005	0.004	0.011	0.007	0.012	0.014	0.006	0.011
$\rho_0 = 4;\ \beta_0 = (1, -1, 1, -1);\ \gamma_0 = (1, 0, 1, -1); \lambda_0 = (-1, -1)$										
S	ρ	β				γ_1	γ_3	γ_4	λ	
	Average Bias									
200	0.108	0.022	-0.022	0.012	-0.006	0.066	0.084	-0.080	-0.017	-0.007
400	0.069	0.013	0.000	-0.004	0.034	0.075	0.092	-0.102	-0.020	-0.011
800	0.033	0.008	0.012	-0.009	0.054	0.075	0.088	-0.111	-0.018	-0.011
	Mean Squared Error									
200	0.118	0.014	0.022	0.019	0.059	0.032	0.053	0.074	0.010	0.022
400	0.064	0.006	0.010	0.009	0.032	0.020	0.032	0.047	0.006	0.011
800	0.034	0.003	0.005	0.005	0.018	0.014	0.021	0.032	0.003	0.006

we normalize the sum of the first row in G to 1. Random utility shocks ε_i are drawn independently from the standard logistic distribution.

instead of asymmetric components such as status. We consider this a reasonable first-order approximation to the institution we study in the next section.

Our experiments follow a 2-by-3-by-3 factorial design: we consider different group sizes $n = 4$ and 6; different sample sizes $S = 200$, 400, and 800; and different true peer effect parameters $\rho_0 = 1$, 2, and 4, while keeping other parameters constant with $(\beta_0, \gamma_0, \lambda_0) = (1, -1, 1, -1; 1, 0, 1, -1; -1, -1)$.[6] For each experimental design, we draw individual characteristics and random utility shocks, calculate influence matrices and simulate decisions in equilibria in samples. We then use the simulated samples to estimate parameters using the nested fixed point maximum likelihood method. We replicate each process for 1000 times, and report the average bias and mean squared errors of our estimators in Table 1 (for $n = 4$) and Table 2 (for $n = 6$).

In all experimental designs, our estimators recover the true parameters quite well. Consistently, as sample sizes (the numbers of groups) increase, the mean squared errors converge roughly at the root-n rate. Also, when we compare the two tables, we see evidence of bigger group sizes markedly improving the performance of our estimator.

7 Peer Effects in Volunteering Activities

In this section we apply our method to investigate the peer effects in college students' decisions to participate in volunteering activities, using a new dataset collected from a university in China.[7]

The exact information structure underlying students' volunteering decisions depends on subtle details such as the relation between roommates

[6] Remember that, according to the model setup, the four entries of β_0 or γ_0 are parameters for $(W_{i1}, W_{i2}, Z_{i1}, Z_{i2})$ respectively, and two entries of λ_0 correspond to (W_{i1}, W_{i2}).

[7] An agreement with the university presents us from disclosing its name.

and the nature of incentives related to volunteering. To a large extent, these institutional details cannot be directly inferred from the dataset alone. In some settings, an assumption of incomplete information is used for modeling simultaneous decisions by individuals that are close to each other: e.g., among family members such as siblings (Hiedemann and Stern, 1999) and couples (Nehring, 2004). In many aspects, college roommates in our application are more likely to have private incentives than family members. We believe the incomplete information setting provides a reasonable first-order approximation to the actual interaction between the roommates.

7.1 Data Description

The dataset we use includes administrative records of demographic information, dormitory room assignment, and volunteering activities of all freshmen students enrolled at the university in 2015. These students also participated in a survey as the class of 2019 graduates in a related survey (AEA RCT Registry number AEARCTR-0004296).[8] In this cohort, a total of 3,982 freshmen were assigned to 955 rooms. University documents showed that the administrators in charge of room assignment were asked to maximize student diversity in each room in terms of majors of study, provinces of origin, and ethnic groups.[9] The university did not admit any international undergraduate students. Among all students in the cohort, 3,569 stayed in the same room throughout four years of college.

[8] The public URL for the trial is: http://www.socialscienceregistry.org/trials/4296

[9] Among these students, 3,781 were ethnic Hans, the ethnic majority in China. Therefore no two minority students were assigned to the same room.

Our sample consists of 1,964 students assigned to 491 four-person rooms.[10] Interviews with students and university administrators suggested that these roommates typically took different classes (because they had different majors) and rarely interacted outside the dormitory setting. For these reasons and for simplicity, we maintain the assumption that the private shocks are drawn independently from the same distribution, and rule out their correlations (e.g., resulting from an outside person interacting with all students in the same room, or students interacting with others living on the same floor).

At the beginning of academic year 2016-17, the university started to use a mobile phone application to keep track of student participation in certain on-campus and off-campus activities. The goal was to collect input data for calculating a "comprehensive performance index" for each student. The index was important for students seeking university scholarships, because scholarship eligibility requires a minimal level of the comprehensive performance index as well as GPA. The index is calculated on a full scale of 200 points: 100 for GPA, and 100 for a measure of "quality development." In an academic year, the students could obtain 5 quality development points for participation in every 20 hours of volunteering activities. These volunteering activities ranged from helping childless elders to assisting the organization of academic conferences.

We keep track of this cohort for two academic years between 2016 and 2018. In 2016-17, 77.2% of these 1,964 students participated in at least one volunteering activity (as measured by a dummy covariate *PrevYrVol*); the percentage dropped to 46.9% in 2017-2018 due to busier schedules

[10] Other room sizes each have less than 200 observations.

(according to our outcome/dependent variable *Volunteer*): most students started preparing for admission exams for domestic or international graduate schools, or the job market. We include four additional covariates: *Age* (measured by birth year and month), *Rural* (whether the student was registered as a member of an "agricultural family" in China's nationwide household registration system), *SingleChild* (whether the student had no siblings), and *Science* (whether the student took a science-based or liberal arts-based format of the national college entrance examination). Table 3 reports the summary statistics.

Table 3: Summary Statistics for the Student Sample

Variable	Mean	Standard Deviation
Age	18.533	0.650
Science	0.512	0.500
Rural	0.321	0.467
SingleChild	0.704	0.457
PrevYrVol	0.772	0.419
Volunteer	0.469	0.499

7.2 Estimation and Counterfactual Analysis

We estimate the social effects in students' binary decisions to participate in volunteering activities during the 2017-2018 academic year. The social influence matrix G is parameterized as in Sections 5 and 6. We assign (*Science*, *Rural*, *SingleChild*) to W_i, and (*Age*, *PrevYrVol*) to Z_i; so G is determined by W_is. Our choice of Z_i to serve the exclusion restriction is based on group discussions with 28 students; they suggested that *Age* and *PrevYrVol* are not significant contributors for what they perceive as the strength of social effects.

We also maintain a working assumption that *Age* has no contextual effect, i.e., $\gamma_{age} = 0$. This is plausible because the roommates in our sample were from the same cohort and it is unlikely that small differences in birth months would have non-trivial exogenous effects.

Table 4 reports the nested fixed point maximum likelihood estimates with standard errors. The estimates for direct effects (β) for *Age*, *Rural*, *SingleChild*, and *PrevYrVol* are all statistically significant with negative, positive, negative, and positive signs respectively. This suggests that, absent contextual and peer effects, a younger student with a rural origin and some siblings, and some volunteering experience in the previous year would be more inclined to volunteer than students with other demographic features. The directions of the effects are consistent with the administrators' impressions.

The estimates for contextual effect coefficients (γ) for *Science* and *SingleChild* are both statistically significant with negative signs; the contextual effect for *PrevYrVol* is statistically positive. These signs are consistent with those of direct individual effects in β; so for these two variables, the results indicate that a student's exposure to an influencer who tends to volunteer increases his/her propensity to volunteer.

In comparison, having a roommate with a rural origin appears to reduce the likelihood to volunteer, even though the direct effect of *Rural* on one's own volunteering decision is positive. One possible explanation is that a rural roommate's choices of activities were viewed as unfashionable and thus shunned. Summarizing contextual effect estimates, we know that, with more roommates from science, rural, and single-child backgrounds, an individual student tends to volunteer less.

Our point estimate of the peer effect coefficient ρ is 0.739, with a 95% confidence interval of $[0.084, 1.394]$; so it is statistically significant, confirming simultaneity and mutual influence between volunteering decisions by peers/roommates. Also, remember that Assumption (MSI) is sufficient but not necessary for a single equilibrium in the sample (Assumption 1). To test Assumption (MSI), note that it requires $|\rho| \leq \frac{1}{(n-1)C \sup_{\varepsilon} f(\varepsilon)}$. But with $n = 4$, $C = \frac{1}{2}$, and $\sup_{\varepsilon} f(\varepsilon) = \frac{1}{\sqrt{2\pi}}$ in the current specification, this upper bound is approximately 1.671, and outside the 95% confidence interval.

Table 4 also reports how W_i affects the influence matrix through the coefficients in λ. The estimated coefficients for (*Science*, *Rural*, *SingleChild*) are $\hat{\lambda} = (-1.078, -1.062, -0.680)$, all of which are statistically significant. These estimates provide evidence for homophily among the students. That is, roommates with similar demographic features tend to have stronger social influence on each other. Moreover, these estimates also demonstrate strong heterogeneity in social effects between students, which would have been lost if we had assumed that the students only influenced each other in a homogeneous fashion (e.g., G assigns equal weights for all group members as in linear-in-means social interaction models).

To highlight the degree of heterogeneity in social effects, we plot in Figure 1 the estimated density of the averages of off-diagonal components in the social influence matrix G across all dormitory rooms. In other words, for each room, we calculate the average of all estimated $\hat{G}_{ij}$ with $i \neq j$ in that room; Figure 1 is the standard kernel density plot for the 491 averages calculated. The

Table 4: Estimation for Volunteering Choices

	Estimate	Standard Errors
Direct Effects (β)		
Age	-0.092**	0.004
Science	-0.050	0.041
Rural	0.299**	0.035
SingleChild	-0.176**	0.026
PrevYrVol	1.718**	0.037
Contextual Effects (γ)		
Science	-0.435**	0.072
Rural	-0.336**	0.098
SingleChild	-0.195**	0.064
PrevYrVol	0.517**	0.189
Influence Matrix Parameters (λ)		
Science	-1.078**	0.235
Rural	-1.062**	0.444
SingleChild	-0.680**	0.232
Peer Effects (ρ)	0.739**	0.334

* and **: 10% and 5% significant.

bell-shaped pattern in the density suggests that rooms with extremely low or high average social influences are rare.

Heterogeneity of social effects is also present within dormitories: Figure 2 plots the density of standard deviations between off-diagonal components of G across all rooms. That is, within each room, the standard deviation of all estimated $\hat{G}_{ij}$ with $i \neq j$ is used, and Figure 2 is the standard kernel density plot for all the standard deviations. This density also roughly demonstrates a bell shape.

Our structural approach is of particular interest for policymakers, because it can be used to analyze how different schemes of roommate assignments would affect student participation in volunteering activities. To illustrate, we conduct

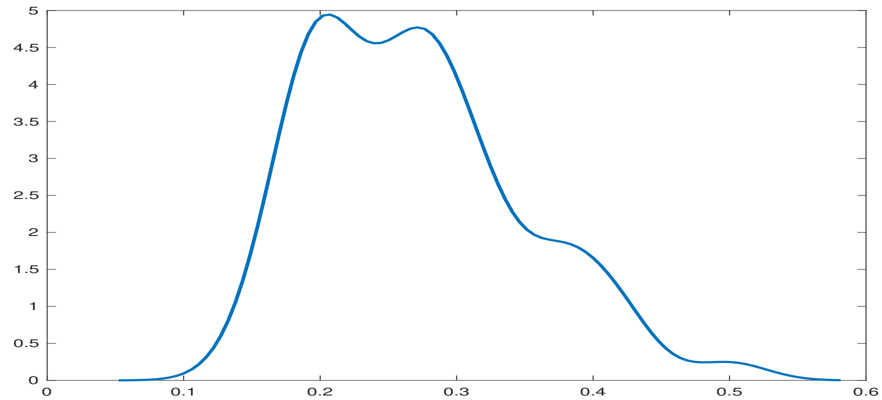

Figure 1: Density of the Averages of Estimated Social Influences across Different Rooms

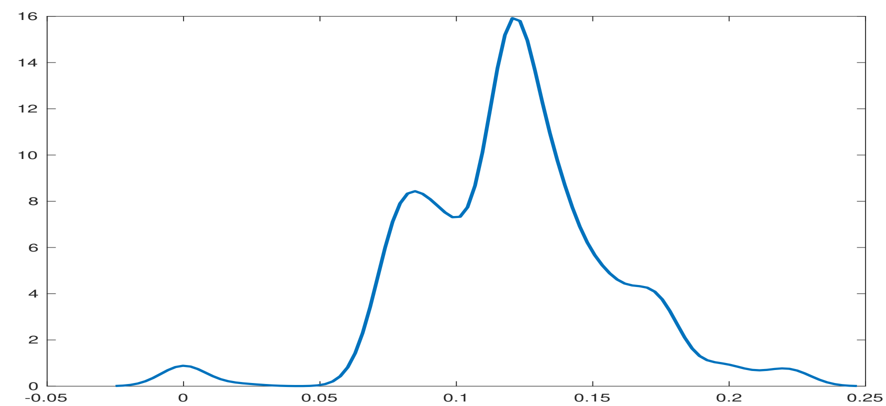

Figure 2: Density of the Standard Deviations of Estimated Social Influences across Different Rooms

counterfactual exercises which calculate the numbers of predicted volunteering activities when students are assigned to rooms under different schemes. We follow a 2-by-3 factorial design. Based on three different covariates *Science*, *Rural*, and *SingleChild*, we either form maximally mixed dormitory rooms or maximally segregated ones, from the given pool of students.

For example, 51.2% of our sample took a science-based format of the national college entrance examination. Therefore, when we conduct the counterfactual analysis with regard to *Science*, we generate two counterfactual datasets: in the first dataset, each room has two science students and two non-science students; in the second dataset, half of the rooms (245) only have science students and the other half only have non-science students. The distribution of all other covariates are the same as in the real sample. We draw utility shocks, and calculate the numbers of volunteering students in the two datasets.

We also follow the procedure above for the other two characteristics, *Rural* and *SingleChild*. Table 5 reports the results for all designs. Across each of the three counterfactual scenarios, mixing generally leads to marginally fewer incidences of volunteering.[11]

Table 5: Counterfactual Analysis for Volunteering Choices

Volunteering out of 1,964 Students		
	Mixing	Segregating
Science	955	1008
Rural	972	1002
SingleChild	957	987

[11] Even when peer effect coefficient ρ is known to be positive, the exercise is nontrivial, and the result can go both ways. Basically, negative λ implies that mixing leads to weaker strengths in the links, and because of the nonlinearity of our model, weaker link strengths (smaller entries in G) do not necessarily translate into lower volunteering probabilities.

8 Concluding Remarks

We identify and estimate heterogeneous social effects in scenarios where modeling these effects as homogeneous is inadequate, and the extent to which a person's choice is influenced by another (or whether it is influenced at all) is unobserved to the researcher. We apply our method to analyze social effects in college students' volunteering activities, and find evidence for heterogeneity in these effects as well as homophily in social influence.

We conclude with some discussions about open questions and directions for future research. First, the inference of information structure in social networks per se is an interesting open question. This might be possible under stronger parametric restrictions on unobservable errors, e.g., as in Grieco (2014).

Second, our paper does not study heterogeneous social effects in *complete* information settings. When payoff shocks are commonly known to all group members, the link between model elements and equilibrium outcome is different from, and more complex than, our current setting with private shocks. This is because a model of simultaneous choices under complete information generally admits multiple Nash equilibria. As a result, our identification method does not apply in that case.

Third, our paper does not address multiple equilibria or unobserved group fixed effects in the sample; but they may be appropriate modeling choices in some other empirical settings. Xiao (2018) and Aguirregabiria and Mira (2019) estimated Bayesian games with multiple equilibria and unobserved heterogeneity at the game level. Their methods treat equilibrium selection as a source of unobserved heterogeneity, and start with an eigenvalue decomposition that recovers the CCPs conditional on unobserved heterogeneity and equilibrium selection. In principle, a similar first step may be useful for accommodating multiple equilibria and unobserved heterogeneity in binary choices over a social network with private information. This would require partitioning a group into two subsets so that an eigenvalue decomposition of the joint distribution of choices by these subsets is feasible under appropriate rank and invertibility conditions. Once the choice probabilities conditional on equilibrium selection and unobserved heterogeneity are recovered, we can restore Equation (4) with $\mathbf{p}(X)$ being replaced by these CCPs. Our method can then be adapted to identify and estimate the model. This is a major departure from our model, and the aforementioned eigenvalue decomposition step has to be motivated by more primitive conditions and implemented in estimation with further technical details. This is beyond the scope of our paper, and could be a promising direction for future research.

REFERENCES

Aguirregabiria, V. and P. Mira (2019). Identification of games of incomplete information with multiple equilibria and unobserved heterogeneity. *Quantitative Economics 10*(4), 1659–1701.

Aradillas-Lopez, A. (2010). Semiparametric estimation of a simultaneous game with incomplete information. *Journal of Econometrics 157*(2), 409–431.

Aradillas-Lopez, A. (2012). Pairwise-difference estimation of incomplete information games. *Journal of Econometrics 168*(1), 120–140.

Athey, S. (2001). Single crossing properties and the existence of pure strategy equilibria in games of incomplete information. *Econometrica 69*(4), 861–

889.

Bajari, P., H. Hong, J. Krainer, and D. Nekipelov (2010). Estimating static models of strategic interactions. *Journal of Business & Economic Statistics 28*(4), 469–482.

Blume, L. E., W. A. Brock, S. N. Durlauf, and R. Jayaraman (2015). Linear social interactions models. *Journal of Political Economy 123*(2), 444–496.

Boucher, V. and A. Houndetoungan (2020). Estimating peer effects using partial network data. *Working Paper*.

Bramoullé, Y., H. Djebbari, and B. Fortin (2009). Identification of peer effects through social networks. *Journal of Econometrics 150*(1), 41–55.

Bresnahan, T. F. and P. C. Reiss (1991). Empirical models of discrete games. *Journal of Econometrics 48*(1-2), 57–81.

Brock, W. A. and S. N. Durlauf (2001). Discrete choice with social interactions. *The Review of Economic Studies 68*(2), 235–260.

Brock, W. A. and S. N. Durlauf (2007). Identification of binary choice models with social interactions. *Journal of Econometrics 140*(1), 52–75.

De Paula, Á., I. Rasul, and P. Souza (2019). Identifying network ties from panel data: Theory and an application to tax competition. *Working Paper*.

De Paula, Á. and X. Tang (2012). Inference of signs of interaction effects in simultaneous games with incomplete information. *Econometrica 80*(1), 143–172.

De Paula, Á. and X. Tang (2020). Testable implication of multiple equilibria in bayesian games with correlated types. *Working Paper, Rice University & University College London*.

Florens, J.-P. and E. Sbaï (2010). Local identification in empirical games of incomplete information. *Econometric Theory*, 1638–1662.

Glaeser, E. and J. A. Scheinkman (2000). *Non-market interactions*. Advances in Economics and Econometrics: Theory and Applications, Eight World Congress. Cambridge University Press.

Graham, B. S. (2008). Identifying social interactions through conditional variance restrictions. *Econometrica 76*(3), 643–660.

Graham, B. S. and J. Hahn (2005). Identification and estimation of the linear-in-means model of social interactions. *Economics Letters 88*(1), 1–6.

Grieco, P. L. (2014). Discrete games with flexible information structures: An application to local grocery markets. *The RAND Journal of Economics 45*(2), 303–340.

Heckman, J. J. (1976). The common structure of statistical models of truncation, sample selection and limited dependent variables and a simple estimator for such models. In *Annals of Economic and Social Measurement, Volume 5, Number 4*, pp. 475–492. NBER.

Hiedemann, B. and S. Stern (1999). Strategic play among family members when making long-term care decisions. *Journal of Economic Behavior & Organization 40*(1), 29–57.

Horst, U. and J. A. Scheinkman (2006). Equilibria in systems of social interactions. *Journal of Economic Theory 130*(1), 44–77.

Hu, Y. and Z. Lin (2019). Misclassification and the hidden peer effects in attitude. *Working Paper*.

Krauth, B. V. (2006). Simulation-based estimation of peer effects. *Journal of Econometrics 133*(1), 243–271.

Lee, L.-f. (2007). Identification and estimation of econometric models with group interactions, contextual factors and fixed effects. *Journal of Econometrics 140*(2), 333–374.

Lee, L.-f., J. Li, and X. Lin (2014). Binary choice models with social network under heterogeneous rational expectations. *Review of Economics and Statistics 96*(3), 402–417.

Lewbel, A., X. Qu, and X. Tang (2019). Social networks with misclassified or unobserved links. *Working Paper*.

Lewbel, A. and X. Tang (2015). Identification and estimation of games with incomplete information using excluded regressors. *Journal of Econometrics 189*(1), 229–244.

Li, T. and L. Zhao (2016). A partial identification subnetwork approach to discrete games in large networks: An application to quantifying peer effects. *Working Paper*.

Lin, Z. (2019). Social interactions quantiles. *Working Paper*.

Lin, Z. and H. Xu (2017). Estimation of social-influence-dependent peer pressure in a large network game. *The Econometrics Journal 20*(3), S86–S102.

Liu, X. (2019). Simultaneous equations with binary outcomes and social interactions. *Econometric Reviews 38*(8), 921–937.

Manski, C. F. (1993). Identification of endogenous social effects: The reflection problem. *The Review of Economic Studies 60*(3), 531–542.

Nehring, K. (2004). The veil of public ignorance. *Journal of Economic Theory 119*(2), 247–270.

Newey, W. K. and D. McFadden (1994). Large sample estimation and hypothesis testing. *Handbook of Econometrics 4*, 2111–2245.

Patacchini, E., E. Rainone, and Y. Zenou (2017). Heterogeneous peer effects in education. *Journal of Economic Behavior & Organization 134*, 190–227.

Pesendorfer, M. and P. Schmidt-Dengler (2008). Asymptotic least squares estimators for dynamic games. *The Review of Economic Studies 75*(3), 901–928.

Rust, J. (1987). Optimal replacement of GMC bus engines: An empirical model of harold zurcher. *Econometrica 55*(5), 999–1033.

Soetevent, A. R. and P. Kooreman (2007). A discrete-choice model with social interactions: With an application to high school teen behavior. *Journal of Applied Econometrics 22*(3), 599–624.

Tamer, E. (2003). Incomplete simultaneous discrete response model with multiple equilibria. *The Review of Economic Studies 70*(1), 147–165.

Wan, Y. and H. Xu (2014). Semiparametric identification of binary decision games of incomplete information with correlated private signals. *Journal of Econometrics 182*(2), 235–246.

Wooldridge, J. M. (2010). *Econometric analysis of cross section and panel data*. MIT Press.

Xiao, R. (2018). Identification and estimation of incomplete information games with multiple equilibria. *Journal of Econometrics 203*(2), 328–343.

Xu, H. (2018). Social interactions on large networks: A game theoretic approach. *International Economic Review 59*(1), 257–284.

Appendix A Proofs

A.1 Proof of Lemma 1

Recall that $\mu^{(k)} := \beta_k^z I + \gamma_k^z G$. By construction, $a\beta_1^z + b\beta_2^z = 0$ and $a\gamma_1^z + b\gamma_2^z = \rho$. Thus, the left-hand side of Equation (6) equals $0 + \rho G := \mu^{(0)}$.

To see the uniqueness of the solution, consider $(\tilde{a}, \tilde{b}) \neq (a, b)$, and denote $c := \tilde{a}\beta_1^z + \tilde{b}\beta_2^z$ and $d := \tilde{a}\gamma_1^z + \tilde{b}\gamma_2^z$. Because (β^z, γ^z) has full rank, $(c, d) \neq (0, \rho)$. Next, note that the diagonal entries in G and the off-diagonal entries in I are zeros. Hence,

$$\tilde{a}\mu^{(1)} + \tilde{b}\mu^{(2)} = cI + dG \neq \rho G$$

and Equation (6) does not hold for such $(\tilde{a}, \tilde{b})$. □

A.2 Proof of Theorem 2

The proof is similar to that of Theorem 2.1 in Newey and McFadden (1994).

Under the identification condition, $L_0(c)$ is uniquely maximized at θ_0. By the definition of MLE, for any $\epsilon > 0$, we have $\hat{L}_S(\hat{\theta}) > \hat{L}_S(\theta_0) - \frac{1}{3}\epsilon$ with probability approaching one (w.p.a.1). By the law of large number, we have that $L_0(\hat{\theta}) > \hat{L}_S(\hat{\theta}) - \frac{1}{3}\epsilon$ and $\hat{L}_S(\theta_0) > L_0(\theta_0) - \frac{1}{3}\epsilon$.

Therefore, w.p.a.1,

$$L_0(\hat{\theta}) > \hat{L}_S(\hat{\theta}) - \frac{1}{3}\epsilon > \hat{L}_S(\theta_0) - \frac{2}{3}\epsilon > L_0(\theta_0) - \epsilon. \tag{11}$$

Let $\mathcal{N}$ be any open subset of Θ containing θ_0. Since $\Theta \cap \mathcal{N}^c$ is compact, θ_0 uniquely maximizes $L_0(\theta)$, and $L_0(\theta)$ is continuous, we have

$$\sup_{\theta \in \Theta \cap \mathcal{N}^c} L_0(\theta) = L_0(\theta^*) < L_0(\theta_0).$$

Therefore, for $\epsilon = L_0(\theta_0) - \sup_{\theta \in \Theta \cap \mathcal{N}^c} L_0(\theta)$, we have w.p.a.1,

$$L_0(\hat{\theta}) > \sup_{\theta \in \Theta \cap \mathcal{N}^c} L_0(\theta).$$

Hence, $\hat{\theta} \in \mathcal{N}$.

By the definition of $\hat{\theta}$, we have $\frac{\partial \hat{L}_S(\hat{\theta})}{\partial \theta} = 0$. Taylor expansion gives us

$$\frac{\partial \hat{L}_S(\theta_0)}{\partial \theta} + \frac{\partial^2 \hat{L}_S(\tilde{\theta})}{\partial \theta \partial \theta'}(\hat{\theta} - \theta_0) = 0,$$

where $\tilde{\theta}$ is between $\hat{\theta}$ and θ_0. Since we have independent repeated small blocks $s = 1, \cdots, S$, a standard central limit theorem applies to $\frac{\partial \hat{L}_S(\theta_0)}{\partial \theta}$; by a classical information equality, we have

$$\sqrt{S}(\hat{\theta} - \theta_0) \xrightarrow{d} N(0, \Omega(\theta_0)^{-1}).$$ □