Exclusion Restrictions in Dynamic Binary Choice Panel Data Models *

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We revisit the use of *exclusion restrictions* in the semiparametric binary choice panel data model with predetermined regressors introduced in Honoré and Lewbel (2002). The identification strategy in Honoré and Lewbel (2002) requires the restriction (Assumption A.2, p. 2055) that one of the explanatory variables (which we refer to as an "excluded regressor" henceforth) is independent of the individual fixed effect and time-varying idiosyncratic errors, conditional on the other regressors. Their model is presented in a framework where the explanatory variables are predetermined, including lagged dependent variables. We show that in a dynamic binary choice panel data model, when excluded regressors (ER) are serially correlated, the A.2 condition in Honoré and Lewbel (2002) does not always hold and this condition was crucial for the identification and consistent estimation of their model..

We show how the exclusion restriction in Assumption A.2 of Honoré and Lewbel (2002) could fail in a dynamic setting, by considering the model:

$$y_{it} = I[v_{it} + x_{it}'\beta_0 + \alpha_i + \epsilon_{it} \geq 0] \tag{1.1}$$

where $i = 1, 2, ..., n$, and $t = 1, 2, ..., T$. Here $I[\cdot]$ is the indicator function that equals one if "$\cdot$" is true and zero otherwise, $v_{it} \in \mathbb{R}$ is the excluded regressor whose coefficient is normalized to one, x_{it} is a vector of other regressors (possibly predetermined), β_0 is a vector of coefficients, α_i is an individual-specific fixed effect, and the distribution of ϵ_{it} is unknown.

Honoré and Lewbel (2002) identify the parameters in this model under an exclusion restriction that $\alpha_i + \epsilon_{it}$ is independent of v_{it}, conditional on x_{it}. In a dynamic panel data model, one of the components in x_{it} is the lagged dependent variable y_{it-1}, which itself is a function of α_i, ϵ_{it-1}, x_{it-1} and v_{it-1}. As a result, serial correlation in v_{it} leads to a complex dependence structure between y_{it-1}, x_{it}, v_{it} and $\alpha_i + \epsilon_{it}$, which is generally not compatible with the exclusion restriction (Assumption A.2) in Honoré and Lewbel (2002). This is true even if v_{it} is independent of α_i and ϵ_{it} conditional on the other components in x_{it} in each period.

To illustrate our main idea, consider a simplified version of the model in (1.1) with two periods following the first observed dependent variable y_{i0} ($T = 2$) and only two explanatory variables, which consist of an excluded regressor $v_{it} \in \mathbb{R}$ and the lagged dependent variable y_{it-1}:

$$y_{it} = I[v_{it} + y_{it-1}\gamma_0 + \alpha_i + \epsilon_{it} \geq 0] \text{ for } t = 1, 2, \tag{1.2}$$

where the parameter of interest is γ_0. Let $\mathbf{y}_i \equiv (y_{i0}, y_{i1}, y_{i2})$, $\mathbf{v}_i \equiv (v_{i1}, v_{i2})$ and $\epsilon_i \equiv (\epsilon_{i1}, \epsilon_{i2})$. We assume:

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[EM1] (*Random Sampling*) For each cross-sectional unit i, the vector $(\mathbf{y}_i, \mathbf{v}_i, \alpha_i, \epsilon_i)$ is independently drawn from the same data-generating process. The vector $(\mathbf{y}_i, \mathbf{v}_i)$ is observed while (α_i, ϵ_i) is not.

Honoré and Lewbel (2002) state the following exclusion restriction (on p.2055) on the model in (1.1) for identification and estimation:

ASSUMPTION A.2: For $t = 1, 2$, $\alpha_i + \epsilon_{it}$ is independent of v_{it} conditional on x_{it} and z_i.

Note that Honoré and Lewbel (2002) state this assumption by conditioning on an instrument z_i, which may overlap with the exogenous variables. For simplicity, suppose that the value y_{i0} is degenerate at 0 in the data-generating process, and to further ease notation, in what follows we suppress z_i. Assumption A.2 in Honoré and Lewbel (2002) requires

$$(\alpha_i + \epsilon_{it}) \perp v_{it} \text{ conditional on } y_{it-1} \text{ for } t = 1, 2. \tag{1.3}$$

We show that, if v_{i1}, v_{i2} are serially correlated as in our example below, then (1.3) does not always hold even when $(\alpha_i + \epsilon_{i1}, \alpha_i + \epsilon_{i2})$ is independent of (v_{i1}, v_{i2}). To simplify notation, we drop the subscript i for all random variables, and let $e_t \equiv -(\alpha + \epsilon_t)$ for $t = 1, 2$. Assume that (e_1, e_2) is independent of (v_1, v_2); and that the joint distribution of (e_1, e_2) and the joint distribution of (v_1, v_2) are both exchangeable in the index $t \in \{1, 2\}$. [1] Let F (and f) denote the marginal distribution (and density) of v_t; let G (and g) denote the marginal distribution (and density) of e_t. Define $F(s'|s) \equiv \Pr(v_1 \leq s'|v_2 = s)$ and $G(r'|r) \equiv \Pr(e_1 \leq r'|e_2 = r)$. That F, G do not vary with $t = 1, 2$ is a consequence of the exchangeability condition.

We will show that e_2 is not independent of v_2 conditional on $y_1 = 1$ (or equivalently, $v_1 - e_1 \geq 0$) if v_t is serially correlated between $t = 1, 2$. By definition,

$$\begin{aligned}
&\left.\frac{\partial^2 \Pr(e_2 \leq \tilde{r}, v_2 \leq \tilde{s}|y_1 = 1)}{\partial \tilde{r} \partial \tilde{s}}\right|_{\tilde{r}=r, \tilde{s}=s} \\
= &\left.\frac{\partial^2}{\partial \tilde{r} \partial \tilde{s}} \left(\frac{\Pr(v_1 - e_1 \geq 0, e_2 \leq \tilde{r}, v_2 \leq \tilde{s})}{\Pr(v_1 - e_1 \geq 0)} \right)\right|_{\tilde{r}=r, \tilde{s}=s} = \frac{\Pr(v_1 - e_1 \geq 0|e_2 = r, v_2 = s) g(r) f(s)}{\Pr(v_1 - e_1 \geq 0)} \\
= &\frac{g(r) f(s) \int \Pr(e_1 \leq \tilde{s}|v_1 = \tilde{s}, e_2 = r, v_2 = s) dF(\tilde{s}|s)}{\int G(\tilde{s}) dF(\tilde{s})} = \frac{g(r) f(s) \int G(\tilde{s}|r) dF(\tilde{s}|s)}{\int G(\tilde{s}) dF(\tilde{s})}.
\end{aligned}$$

where the third and fourth equalities follow from an application of the Law of Total Probability in the numerator and the denominator, and from the independence between (e_1, e_2) and (v_1, v_2). On the other hand,

$$\begin{aligned}
\left.\frac{\partial \Pr(e_2 \leq \tilde{r}|y_1 = 1)}{\partial \tilde{r}}\right|_{\tilde{r}=r} &= \left.\frac{\partial}{\partial \tilde{r}} \left[\frac{\Pr(v_1 - e_1 \geq 0, e_2 \leq \tilde{r})}{\Pr(v_1 - e_1 \geq 0)} \right]\right|_{\tilde{r}=r} = \frac{\Pr(v_1 - e_1 \geq 0|e_2 = r) g(r)}{\Pr(v_1 - e_1 \geq 0)} \\
&= \frac{g(r) \int \Pr(e_1 \leq \tilde{s}|v_1 = \tilde{s}, e_2 = r) dF(\tilde{s})}{\int G(\tilde{s}) dF(\tilde{s})} = \frac{g(r) \int G(\tilde{s}|r) dF(\tilde{s})}{\int G(\tilde{s}) dF(\tilde{s})}
\end{aligned}$$

and

$$\begin{aligned}
\left.\frac{\partial \Pr(v_2 \leq \tilde{s}|y_1 = 1)}{\partial \tilde{s}}\right|_{\tilde{s}=s} &= \left.\frac{\partial}{\partial \tilde{s}} \left[\frac{\Pr(v_1 - e_1 \geq 0, v_2 \leq \tilde{s})}{\Pr(v_1 - e_1 \geq 0)} \right]\right|_{\tilde{s}=s} = \frac{\Pr(v_1 - e_1 \geq 0|v_2 = s) f(s)}{\Pr(v_1 - e_1 \geq 0)} \\
&= \frac{f(s) \int \Pr(e_1 \leq \tilde{s}|v_1 = \tilde{s}, v_2 = s) dF(\tilde{s}|s)}{\int G(\tilde{s}) dF(\tilde{s})} = \frac{f(s) \int G(\tilde{s}) dF(\tilde{s}|s)}{\int G(\tilde{s}) dF(\tilde{s})},
\end{aligned}$$

[1] Exchangeability is not assumed in Honoré and Lewbel (2002), but we introduce it here to demonstrate how Assumption A2 could be violated.

where the last two equalities hold because of similar reasons. Thus for all (e, v),

$$\frac{\left(\frac{\partial^2 \Pr(e_2 \le \tilde{r}, v_2 \le \tilde{s} | y_1 = 1)}{\partial \tilde{r} \partial \tilde{s}} \Big|_{\tilde{r}=r, \tilde{s}=s} \right)}{\left(\frac{\partial \Pr(e_2 \le \tilde{r} | y_1 = 1)}{\partial \tilde{r}} \Big|_{\tilde{r}=r} \right) \left(\frac{\partial \Pr(v_2 \le \tilde{s} | y_1 = 1)}{\partial \tilde{s}} \Big|_{\tilde{s}=s} \right)} = \left(\frac{\int G(\tilde{s}|r) dF(\tilde{s}|s)}{\int G(\tilde{s}|r) dF(\tilde{s})} \right) \left(\frac{\int G(\tilde{s}) dF(\tilde{s})}{\int G(\tilde{s}) dF(\tilde{s}|s)} \right). \tag{1.4}$$

The right-hand side (r.h.s.) of (1.4) is 1 whenever v_1 and v_2 are serially independent, namely $F(s'|s) = F(s')$ for all (s', s) on the joint support of (v_1, v_2). However, if v_1 and v_2 are serially dependent with the r.h.s. of (1.4) not equal to 1, then Assumption A.2 in Honoré and Lewbel (2002) does not hold. To see this, consider an extreme case where v_t has perfect serial correlation (v_t is time-invariant with $\Pr(v_1 = v_2) = 1$). Then the right-hand side of (1.4) becomes:

$$\frac{G(s|r)}{\int G(\tilde{s}|r) dF(\tilde{s})} \frac{\int G(\tilde{s}) dF(\tilde{s})}{G(s)}.$$

Because $G(.|r)$ varies with r due to the dependence between e_1 and e_2, this expression is in general not equal to 1 for all (s, r) on the support of (e_1, e_2). Thus when the excluded regressors are serially correlated in a dynamic binary choice panel data model, the Assumption A.2 in Honoré and Lewbel (2002) does not always hold. It is worth noting that if the model in (1.2) is static (with $\gamma_0 = 0$), then Assumption A.2 holds when $\alpha_i + \varepsilon_{it}$ is independent of v_{it} for $t = 1, 2$. In such a static model, the estimator in Honoré and Lewbel (2002) is consistent even when v_{it} is serially correlated.

Our finding has important implications. First, time invariance in excluded regressors is not permitted by the assumption A.2 in Honoré and Lewbel (2002) when the predetermined regressors include the lagged dependent variable. The other implication pertains to the initial condition. Suppose the initial condition is modeled as

$$y_{i0} = I[\alpha_i + v_{i0} + \epsilon_{i0} > 0] \tag{1.5}$$

where v_{i0} and y_{i0} are reported in the data [2]. If v_{it} is serially independent conditional on v_{i0}, then A.2 in Honoré and Lewbel (2002) holds. On the other hand, if v_{i0} is unobserved in the data and not conditioned on, then A.2 fails in general because of serially correlated excluded regressors.[3] Constructively, we propose a method to allow for serial correlation in v_{it}.

Our method is based on adopting the approach in Honoré and Lewbel (2002) after modeling and accounting for any serial dependence in v_{it}. For example, assume v_{it} follows an AR(1) process: $v_{it} = \rho_0 v_{it-1} + u_{it}$ where ρ_0 is an AR(1) coefficient and u_{it} is unobserved noise. Assume u_{it} is independent of $(\alpha_i, \epsilon_{it})$ conditional on (v_{it-1}, y_{it-1}). (This holds, for example, if u_{it} is independent of $(u_{is})_{s<t}$, α and $(\epsilon_{is})_{s \le t}$). Then we can rewrite the model for $t = 1, 2$ as

$$y_{it} = I[u_{it} + \rho_0 v_{i,t-1} + \gamma_0 y_{it-1} + \alpha_i + \epsilon_{it} > 0]$$

[2] Honoré and Kyriazidou (2000) assume y_{i0} is observed, but not the initial value of regressors. Their identification result is based on independence between ϵ_{it} and all covariates besides lagged y_{it}, an i.i.d. assumption on ϵ_{it}, and an additional period of observed data.

[3] To see this, note that $F_{e_1,e_2|v_1,v_2,y_0=1}(t_1, t_2) = \int F_{e_1,e_2|e_0 \le \tilde{v}}(t_1, t_2) dF_{V_0|v_1,v_2}(\tilde{v})$. The r.h.s. depends on v_1, v_2 due to serial correlation in excluded regressors, even when the series v_t is independent of the series e_t.

Letting u_{it} play the role of the excluded variable in Honoré and Lewbel (2002), and letting $h(u_{it}|v_{it-1}, y_{it-1})$ denote the conditional density of u_{it} given v_{it-1}, y_{it-1}, we note

$$\begin{aligned} & E\left[\frac{y_{it} - I[u_{it} > 0]}{h(u_{it}|v_{it-1}, y_{it-1})}\bigg| v_{it-1}, y_{it-1}\right] \\ = \quad & E\left\{E\left[\frac{I[u_{it} + \rho_0 v_{it-1} + \gamma_0 y_{it-1} + \alpha_i + \epsilon_{it} > 0] - I[u_{it} > 0]}{h(u_{it}|v_{it-1}, y_{it-1})}\bigg| v_{it-1}, y_{it-1}, \alpha_i, \epsilon_{it}\right]\bigg| v_{it-1}, y_{it-1}\right\} \end{aligned}$$

By the same arguments as in Honoré and Lewbel (2002), the inner expectation above simplifies to

$$\rho_0 v_{it-1} + \gamma_0 y_{it-1} + \alpha_i + \epsilon_{it},$$

Now, defining $y^*_{it} \equiv \frac{y_{it} - I[u_{it} > 0]}{h(u_{it}|v_{it-1}, y_{it-1})} - \rho_0 v_{it-1}$, we have

$$E\left(y^*_{it}|v_{it-1}, y_{it-1}\right) = \gamma_0 y_{it-1} + E\left(\alpha_i + \epsilon_{it}|v_{it-1}, y_{it-1}\right).$$

Hence $E(y^*_{it} z_i) = E(y_{it-1} z_i)\gamma_0 + E((\alpha_i + \epsilon_{it}) z_i)$, where here z_i denotes a vector of instruments as in Honoré and Lewbel (2002). Taking the difference of both sides of the equation for consecutive periods allows us to identify γ_0.[4]

References

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[4]Our argument is based on ρ_0 being known, but we note it also can be identified.